the Revista Mensual Climatologica, and the Revista Cientifica Mexicana, and the Ministerio de Fomento, Mexico.

An obituary notice of the late Wm. E. DuBois, was, by appointment, read by Mr. Robert Patterson.*

Dr. Brinton explained to the Society the substance of his paper on the Gods in the Kiché Myth, the Popol Vuh.

Mr. Lesley read Dr. Newberry's paper on the Origin of the Lake Basins, and then remarked on the relation of Dr. Newberry's claims to Prof. Spencer's discoveries and views.†

Mr. Lesley gave a short sketch of the history and progress of the excavations at Assos during the last few months, under the auspices of the Boston Archæological Society, as he obtained it in conversations with Prof. W. R. Ware of Columbia College.

The minutes of the last meeting of the Board of Officers and members in Council were read, and the consideration of the resolution therein was postponed for the next meeting.

Certain valuable manuscripts were ordered, on motion of Dr. Brinton, to be placed by the Library Committee in the custody of the Fidelity for safe keeping.

Pending nomination No. 935, and new nominations Nos. 946 to 950 were read, and the meeting was adjourned.

Photodynamic Notes, IV. By Pliny Earle Chase, LL.D.

(Read before the American Philosophical Society, November 4, 1881.)

91. *Photodynamic Determination of Sun's Mass and Distance.*

In Notes 5 and 23, I estimated Sun's mass both from projectile and from simple oscillatory considerations. In the former note I deduced the distance from an assumed solar density, instead of taking the ratio of variability $d^3 \propto mt^2$. My conviction of the importance of Fourier's theorem has been strengthened by further study, and I accordingly give, in the present note, the coördinate photodynamic elements which may be simply deduced from it.

If we regard the luminiferous æther as a nebulous elastic atmosphere, and the solar system as a partially condensed nebula, the nebula is not homogeneous. It contains, in addition to various subordinate and com-

* This paper will be printed in Vol. xx, No. 111.

† Dr. Newberry's paper will be printed in Vol. xx, No. 111.

paratively unimportant nuclei, two principal and controlling nuclei of nearly equal density, viz.: Sun, at the principal centre of nucleation, and Jupiter at the mean nebular centre between Uranus at apojove, and Neptune at perijove.

The actions and reactions between these controlling nuclei have produced an intermediate maximum condensation in the belt of dense planets, with a principal mass, Earth, near the centre of the belt. The solar radiations are propagated with the velocity of light, v_λ. If we designate Earth's mass by m_3, the actions and reactions of photodynamic *vis viva* at the centre of density, $m_3 v_\lambda^2$, produce gravitating tendencies towards the linear centre of gravity ($\frac{1}{2}$), the centre of linear oscillation ($\frac{1}{3}$), and the centre of conical oscillation ($\frac{1}{4}$), which vary as the fourth power* of the tendencies to orbital velocity. These tendencies are all satisfied by a reacting mass, $m_3 = (\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4})^4 m_0 = \frac{1}{331776}$ of Sun's mass. The photodynamic character of the oscillatory *vis viva* at the centre of condensation is shown by the following proportions :

$$
\begin{array}{lllll}
(1) & m_0 v_0^2 & : m_3 v_\lambda^2 & :: m_3 v_\lambda^2 & : m_5 v_0^2 \\
(2) & v_\lambda & : v_0 & :: v_0 & : v_n \\
(3) & v_n^4 & : v_3^4 & :: p_3 & : p_0
\end{array}
$$

In these proportions m_0 is taken as the unit of mass ; Jupiter's mass, $m_5 = \frac{1}{1047.879}$; v_n = orbital velocity at centre of condensation ; v_0 is a mean proportional between v_λ and v_n ; p_0 = ratio of mean photodynamic projection of Sun's centre from the centre of gravity of Sun and Jupiter, Sun's semidiameter being the unit ; p_3 = mean photodynamic projection of the centre of condensation from Earth's semi-axis major as a unit ; these projections, like the gravitating tendencies, vary as the fourth power of orbital velocity.

If we take $t_\lambda = 497.827$ seconds, and $g = 32.088$ feet, the above proportions give

$v_\lambda =$	186381.4	miles =	299943.5	kilometres.
$v_0 =$	1841.017	" =	2962.75	"
$v_n =$	18.185	" =	29.265	"
$v_3 =$	18.4735	" =	29.729	"
$p_3 =$	1.015866.			
$p_0 =$	1.06499.†			
$\rho_3 =$	92785700	miles =	149320000	kilometers.
$r_0 =$	432574.9	" =	696143.3	"

$n = \rho_3 \div r_0 = 214.496.$

Sun's parallax = $8''.8094$.

* $g \propto \frac{1}{d^2}$; $v = \sqrt{gr} \propto \sqrt{\frac{1}{d}}$; $\therefore g \propto v^4$.

† Stockwell's estimate (*Smithsonian Contrib.*, 232, p. 38), is ;$\frac{1}{9391726}$ = 1.06467 r_0.

92. *Latent Heat of Steam.*

The maintenance of the primitive velocity of undulation is exemplified in the relation between the projectile energy of evaporation and the energy of solar rotation. The equatorial velocity of solar rotation is $\frac{\pi\, g_0 r_0}{v_\lambda}$. The mean rotating velocity of the locus of the centre of gravity of Sun and Jupiter is, therefore, $v_p = \frac{p_0 \pi\, g_0 r_0}{v_\lambda}$. The centre of gravity itself does not rotate, but its action produces æthereal photodynamic undulations, which are propagated with the velocity v_p, *which is equivalent to the projectile velocity of evaporation,* v_s;

$$v_p = v_s.$$

This equation is so important that it may be well to give the calculation in full:

$m_0 \div m_3 = (2 \times 3 \times 4)^4 = 331776$	5.5208450
1 year $= 31558149$ seconds $= t_1$	7.4991115
$t_3 = 2\,\pi \sqrt{\dfrac{r_3 = 3962.8 \text{ miles}}{g_3 = 32.088 \text{ feet}}} \doteq 5073.6$ seconds	3.7053158
$\nu \;= \rho_3 \div r_3 = \left(\dfrac{t_1^2\, m_0}{t_3^2\, m_3}\right)^{\frac{1}{3}} \doteq 23414.2$	4.3694788
$\rho_3 = \nu\, r_3 \doteq 92{,}785{,}700$ miles	7.9674810
$v_0 = v_\lambda = \rho_3 \div 497.827 \doteq 186381.4$ miles	5.2704025
$p_3 = 1.015866$	.0068364
$p_0 = p_3^4 = 1.06499$	.0273455
$\rho_5 = (p_0 m_0 \div m_5)\, r_0 = 1.06499 \times 1047.879\, r_0 \doteq 1115.98\, r_0$	3.0476566
$n \;= \rho_5 \div 5.202796\, r_0 \doteq 214.496$	2.3314197
$r_0 = \rho_3 \div n \doteq 432574.9$ miles	5.6360613
$\sqrt{g_0 r_0} = 2\,\pi n^{\frac{3}{2}}\, r_0 \div t_1 \doteq 270.557$ miles	2.4322592
$\therefore v_p \doteq v_s \doteq 1.31405$ miles	.1186113

From this equation the latent heat of steam, θ, can be readily found, by the equations

$$v_s = \sqrt{2\,gh}$$

$$\theta = \frac{h}{1389.6}$$

Solving these equations we get

$h \doteq 142.064$ miles $\doteq 750098$ feet	5.8751178
$\theta \doteq 539^\circ.794$ C	2.7322280

The following values of θ have been deduced experimentally:

Favre and Silbermann	535°.77
Andrews	535°.90
Regnault	536°.67
Tyndall	537°.20
Despretz	540°.00
Dulong	543°.00

93. *Internal Energy of Ice.*

According to the kinetic theory of gases, the internal movements of the particles of steam are rectilinear, representing a total *vis viva* of projection, against the uniform resistance of gravity, of about $\frac{4}{3} \times 1389.6\ \theta$ feet $\doteq$ 189.42 miles. While the steam is condensed in the form of water or ice, the internal energies tend to maintain a spherical figure. The resultant oscillations (Note 23), can therefore be represented by a conical pendulum of $\frac{1}{4}$ the height of total projection, or $\frac{1}{3}$ of the projectile *vis viva* of evaporation. This gives, for the virtual fall from incipient ebullition to total congelation, $h_1 = \frac{1}{4}\ h \doteq 47.355$ miles $\doteq 250033$ feet; and for the heat of sphericity $\theta_1 = \frac{1}{3}\ \theta \doteq 179^\circ.931$ C. Deducting 100° for the expansion from the freezing to the boiling point, we have $79^\circ.931$ for the "latent heat" of ice, or the heat which is required to overcome its internal energy. The following values have been deduced experimentally:

Desains and De la Provostaye	79°.25
Black	79°.44
Person	80°.00
Hess	80°.34

94. *Photodynamic Moment of Inertia.*

Much of the difficulty which has been encountered in trying to reconcile the nebular hypothesis with actual planetary arrangements, has arisen from overlooking the difference between nucleal retardation and free orbital revolution. Herschel's doctrine of "subsidence" removes this difficulty, and an examination of the primitive planetary belt, before any of its successive divisions into asteroidal and intra-asteroidal belts, two-planet belts, and single-planet belts, shows the photodynamic influence in a very striking manner. The limit of photodynamic *nucleal* rotation is in the asteroidal belt; the photodynamic limit of "subsidence" rotation is at the solar modulus of light. I have often shown* that Saturn represents the photodynamic centre of inertial moment; in the division of inertia among the several belts, provision has been made for the change of linear into synchronous conical oscillations (4 : 1); for the change of synchronous into orbital oscillations ($t \propto l^{\frac{3}{2}}$); for the ratio of nebular radii to radii of subsidence-collision ($\frac{3}{2}$); and for the relative variability of centripetal and orbital tendencies, ($g \propto v_0{}^4$). Hence we find

$$4^{\frac{3}{2}} \times (\tfrac{3}{2})^4 \times M_0 : m_6 : : V_0 : v_s$$

Substituting the values of V_0 and v_s (Note 92) we get

$$M_0 = 3502.2\ m_6.$$

Bessel's estimate was 3501.6.

95. *Photodynamic Centres of Gravity.*

An interesting approximation is shown by the ratio

$$3^3 \times 5 \times M_0 : m_5 : : V_0 : v_3.$$

* Note 12; *ante*, xviii, 431; *et al.*

The ratio of the foregoing note may be expressed as follows :

$$3^4 M_0 : 2 m_6 : : V_0 : v_3.$$

Combining these ratios we get

$$3 m_5 = 10 m_6.$$

The repetition of the pendulum-ratio, and the simplicity of the harmonic factors make these ratios so suggestive that they seem worthy of further study.

96. *Universal Energy of Light.*

It may be well to state the principal facts which are embodied in notes 90-95, in such a way as to show more clearly the simplicity of the relations of the several physical velocities to the velocity of solar radiation.

$$1. \quad \frac{v_\lambda}{u_3} = \frac{h_0}{h_2} = \frac{M_0}{m_3} = \left(\frac{t_n}{t_a}\right)^2 \times \sqrt{\frac{\varphi_0}{\varphi_3}}$$

$$2. \quad \frac{v_\lambda}{v_3} = \frac{\sqrt{M_0 m_5}}{m_3} = \frac{M_0 t_a}{m_3 t_n} \times \sqrt{\frac{\mu}{\chi}}$$

$$3. \quad \frac{v_\lambda}{v_s} = \frac{3^4 M_0}{2 m_6}$$

$$4. \quad v_\lambda v_r = v_0^2 \times \frac{t_n}{t_a}$$

5. $v_s = v_p$ (Note 92)

6. $v_\lambda = g_0 t_0$.

In these equations, v_r = velocity of equatorial rotation which Sun would have if it were expanded to the locus of a particle which revolves with the circular-orbital velocity v_0 ; by the law of conservation of areas v_r varies inversely as radius, while v_0 varies inversely as the square root of radius ; v_3 = Earth's orbital velocity.

If we assume $M_0 = 328470 m_3$, we find $h_0 = 92476500$ miles ; $v_\lambda = 185760$ miles ; $v_3 = 18.412$ miles ; $u_3 = 2986$ ft. ; $v_s = 6916.2$ ft. The following table shows the accordance between theoretical and observed values :

	Theoretical.	Observed.
1. Boiling point,	99.°98	100°
Combining heat of H_2O,	69319	67616. to 69584*
φ_0	140.65	140 lbs. per sq. in.
2. v_3	18.31	18.41
$\chi \div \mu$	107.38	106.67
3. Latent heat of steam,	536.°374	536.°385†

In these comparisons I have made no allowance for the photodynamic projections which have caused orbital eccentricities, as it may be pre-

* See Note 16.

† Mean of first four estimates in Note 92.

sumed that they have affected different elements in different ways and to different degrees. I still think that the true value of M_0 is about one per cent. greater, and the value of h_0 about $\frac{1}{3}$ of one per cent. greater than here given.

Weber and Kohlrausch demonstrated the importance of photic velocity in electromotive energy, by measuring quantity of electricity; Thomson and Maxwell, by measuring electromotive force; Ayrton and Perry, by measuring electrostatic capacity.

97. *Maximum Density of Water.*

If we let u_4 represent the potential velocity of water at its greatest density, we find that the greatest known velocity of æthereal wave-propagation : the greatest known gravitating velocity : : the corresponding circular-orbital velocity : the potential velocity of water at its greatest density.

$$v_\lambda : v_0 \sqrt{2} : : v_0 : u_4 \qquad (1)$$

Let $\theta = 100^\circ$ C. — water-temperature of maximum density; $\rho_3 =$ Earth's semi-axis major $= nr_0$; $r_0 =$ Sun's semi-diameter. Then

$$v_\lambda = \rho_3 \div 497.827 \qquad (2)$$

$$v_0 = n^{\frac{1}{2}} \times 2\pi \rho_3 \div 1 \text{ year in seconds} \qquad (3)$$

$$u_4 = \sqrt{2gh} = \sqrt{2g\theta \times 1389.6 \div 5280} \qquad (4)$$

If we substitute (2), (3) and (4) in (1), and adopt the British Nautical Almanac estimate, $n = 214.45$, the most satisfactory experimental determinations of θ give the following results:

Authorities.	θ	u_4		ρ_3	
C. Von Neumann	96°.32	.55508	miles.	92,746,800	miles.
Plücker and Geissler	96.20	.55473	"	92,689,100	"
G. Hagen	96.13	.55453	"	92,655,300	"
Joule and Playfair	96.055	.55432	"	92,619,200	"
F. Exner	96.055	.55432	"	92,619,200	"
Despretz	96.00	.55416	"	92,592,700	"
F. Rossetti	95.93	.55396	"	92,558,900	"
H. Kopp	95.92	.55393	"	92,554,100	"
Mean	96.076	.55438	"	92,629,300	"

Whatever value may be adopted for Sun's apparent semi-diameter and for θ, the corresponding value for Earth's semi-axis major may be readily found, since $\rho_3 \propto \sqrt{\frac{\theta}{n}}$.

98. *Electric Mass.*

Many of the foregoing relations seem to show that if the factor M, in electric dimensions, has any proper analogy to the mass-factor in ordinary energy, it should be referred to the particles, or the most minute appreciable portions of luminiferous æther. Professor S. P. Thompson, in the Philosophical Magazine for July, 1881,* makes the following noteworthy suggestion:

"Matter has dimensions [M], Energy [ML^2T^{-2}], and Electricity $\left[M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}\right]$. The latter value is obtained from a consideration of the Law of Coulomb, that $Q \times Q \div L^2 = \text{force} = \left[ML\,T^{-2}\right]$; whence

$$Q = \left[(ML^3T^{-2})^{\frac{1}{2}}\right].$$

But the dimensions of self-attractive matter may be similarly considered by Newton's Law that $- M.M \div L^2 = \text{force} = [M\,L\,T^{-2}]$, whence

$$M = [-L^3\,T^{-2}].$$

And if this value be put in place of M in the dimensions of Q above, we get $Q = \left[\sqrt{-1}\,(L^3\,T^{-2})\right]$, a quantity whose dimensions differ only from those of M in being prefixed by the imaginary quantity $\sqrt{-1}$. This seems to indicate an important relation."

99. *Change of State.*

In the Philosophical Magazine for July, 1881, Professor J. H. Poynting discusses "Change of State; Solid-Liquid." He shows that it follows from his "mode of regarding the subject, that, if in any way the ice can be subjected to pressure while the water in contact with it is not so subjected, then the lowering of the melting point per atmosphere is about $11\frac{1}{2}$ times as great as when both are compressed" (p. 34). In Herschel's hypothesis of nebular "subsidence" a similar action is implied, the nebulous or æthereal atmosphere corresponding to the uncompressed water, and the subsiding particles, under the gravitating pressure, corresponding to the compressed ice. In a former study of potential energy (Proc. Soc. Phil. Amer., xvii, 98), I showed that, under such circumstances, "the increase of radial velocity would be sufficient to produce orbital velocity in the periphery of a stationary nebula, when $\sqrt{n} = \sqrt{2}\,(\sqrt{n}-1)$, and $n = \frac{2}{3-2\sqrt{2}} = 11.656854$." The important bearing of this relation upon planetary positions was also shown at the same time.

100. *Earth's Photodynamic Condensation.*

Let t_λ = time in which solar superficial gravitation would communicate the velocity of light, or time of solar half-rotation; t_5 = Jupiter's orbital time; r_3 = Earth's semi-diameter; $r_s = 338.2183\, r_3$ = Earth's synchronous radius, or distance at which a particle would revolve about Earth, synchronously with Earth's revolution about Sun,

Then $$r_s : r_3 :: t_5 : t_\lambda.$$

This gives 12.81 days for t_λ, or 25.62 days for Sun's rotation, which is $\frac{1}{2}$

*Foot-note page 17.

of one per cent. greater than Laplace's estimate and than the mean value which is indicated by other photodynamic considerations.

101. *Earth's Rigidity.*

Sir William Thomson concluded, from investigations of tidal action and equinoctial precession, that "the earth as a whole is much more rigid than any of the rocks that constitute its upper crust," and that the mean tidal effective rigidity must be greater than that of glass. The inconsistency of this hypothesis with the belief of the internal fluidity of the earth led many to question it. Gen. J. G. Barnard (Smithsonian Contributions, 240) applied the principles of the gyroscope to the explanation of precession, and endeavored to show that no increase in the rate of precession arises from fluidity. Thomson subsequently modified his views (Brit. Assoc. Rep., 1876) in accordance with his theory of vortex-atoms, by considerations based on the quasi-rigidity introduced into a liquid by vortex motion. The stress of the æthereal waves must have an important influence both upon the gyroscopic and upon the vortical tendencies, and the rapidity with which they are propagated may perhaps furnish the substitute for the inconceivable rigidity which seemed to be required in the first discussion of the problem.

102. *Sun's Internal Temperature.*

In Note 58, I gave two photodynamic estimates of solar temperature, the second being 3.07 per cent. greater than the first. The second estimate was based on the hypothesis that the whole mass of the Sun is either fluid or gaseous, so that every particle is continually yielding to tendencies toward Sun's centre, toward the centre of gravity of the solar system and toward the immediate centre of gravity. If the whole mass could be collected at Sun's centre, it would revolve about the centre of gravity of the solar system in less than three hours. but the rotation on the axis which partially compensates for the tendency to revolution, requires about 25.5 days. The orbital motion of the Sun about the centre of its stellar system furnishes a slight additional compensation, but the photodynamic stress seems to be mainly represented by radial oscillations which are synchronous with the orbital revolution which Sun's centre would have if it were free. The potential velocity which represents such radial oscillations, is that which would be acquired by vertical fall through half of the diameter, or perihelion parabolic velocity.

103. *Earth's Internal Temperature.*

The small mass of Earth interposes little opposition to orbital tendencies, and its greatest velocity of axial rotation is only about $\frac{1}{64}$ as great as its solar orbital velocity. If Earth's whole mass was homogeneous, gravity within its mass would vary as distance from centre, and the *mean* tendency to orbital velocity, in reaction against the stress of æthereal undulations, would be represented by a virtual fall through $\frac{1}{4}$ of radius, or 990.7 miles.

This represents a thermal energy of $990.7 \times 5280 \div 1389.6 = 3764.32$ calories, which would be communicated by a temperature of 3764°.32 C., or 6775°.78 F. The temperature of melting rock is estimated by Sir William Thomson (Thomson and Tait's Natural Philosophy, i, App. D., p. 716) at 7000° F. Notwithstanding the crudeness of this approximation, it shows that the temperature which represents orbital reactions against æthereal stress is of the same order of magnitude as that of melted rock, thus corroborating other evidence of the probable fluid condition of the greater portion of our globe.

104. *Barometric Strain.*

The evidence which I presented in 1863 (Proc. Soc. Phil. Amer., ix, 283-8), of cyclical atmospheric strains resulting from the combined stresses of pressure, inertia and elasticity, presents an interesting problem for mathematical analysis. I confined myself to an investigation of mere numerical results; in generalizing and extending them, the following FACTS seem most important:

1. The atmospheric daily variation of solar or terrestrial centripetal inertia is comparatively insignificant.
2. The variation of tangential orbital motion, between noon and midnight, is about $\frac{1}{32}$ of the mean motion; the consequent variation in the moment of inertia is about $\frac{1}{16}$ of the mean moment.
3. The orbital moment of inertia is more than 4000 times as great as the equatorial rotary moment.
4. The combined influences of elasticity and orbital moment of inertia tend to drive the atmospheric particles away from the Earth during the first and third quarters of the day, and towards the Earth during the second and fourth quarters.
5. The sum of the instantaneously varying tendencies reaches a maximum in the middle of each quarter.
6. The ratio of the mean equatorial daily variation of the barometer (above or below its mean altitude), to its mean altitude, corresponds very closely to the ratio of equatorial daily rotation (24,895 miles) to the sum of Earth's synodic daily gravitating reactions against Sun's gravitating action ($86400^2 \times 16.044 \div 5280 = 22683300$ miles).
7. The time of maximum disturbance in each quarter of the day, is delayed about an hour after the middle of the quarter at tropical stations.
8. The magnitude of the disturbances in the morning and afternoon is increased by the atmospheric expansion which is due to solar heat.
9. The magnitude of the disturbances during the night is diminished in such proportion as to maintain the *average* quarter-daily change, which is required by the actions and reactions of elasticity and inertia.
10. The tendency to description of equal areas in equal times, leads approximately to the proportion, especially within the tropics,

$$e^2 : e_1^2 : : r : r_1.$$

In this proportion e represents the daily barometric range; e_1, the annual range; r, the virtual radius of daily inertia, $86400^2\, g \div 2$; r_1, the radius of annual inertia, Earth's semi-axis major.

105. *The St. Helena Test.*

The most extensive and satisfactory intertropical observations that appear to have been published, are those of Gen. Sabine, at St. Helena, in latitude 15°57′S. Ganot's formula, $g = 32.174\ (1 - .00256 \cos. 2\ \varphi)$, gives $g = 32.104$, and $r = 86400^2 \times 16.052 \div 5280 = 22694600$ miles. The mean of five years' observation gives $e = .067$ in.; $e_1 = .135$ in.; $r_1 = \frac{e_1^2\ r}{e^2} = 92138400$ miles. This differs by less than $\frac{2}{3}$ of one per cent. from the greatest value that seems likely to result from the final discussion of the observations of the last transit of Venus, and by less than $\frac{1}{3}$ of one per cent. from the result that has been indicated by some of the recent English discussions. The simplicity of this relation, in the case of our planet, is perhaps due partly to Earth's position at the centre of condensation in the solar system.

106. *Gaseous Diffusion.*

Doebereiner, in his researches on spongy platinum, accidentally used a jar which had a slight crack or fissure. He was surprised to find that the water of the pneumatic trough rose into the jar two and two-third inches in twenty-four hours, although there had been no sensible change in the height of the barometer or the heat of the room. This observation led Graham (*Chemical and Physical Researches*, p. 44; *Phil. Mag.*, and *Pogg. Ann.*, 1833), to the experiment from which he deduced his law of gaseous diffusion: "The diffusion or spontaneous intermixture of two gases in contact is effected by an interchange in position of indefinitely minute volumes of the gases, which volumes are not necessarily of equal magnitude, being, in the case of each gas, inversely proportional to the square root of the density of that gas." He subsequently (*Researches*, p. 88; *Phil. Trans.* 1846, 1849), showed that this was a result of diffusive velocities varying "inversely as the square root of their densities," referring also to "the theoretical law of the passage of gases into a vacuum, according to the well-known theorem that the molecules of a gas rush into a vacuum with the velocity they would acquire by falling from the summit of an atmosphere of the gas of the same density throughout; while the height of such an atmosphere, composed of different gases, is inversely as their specific gravities. This is a particular case of the general law of the movement of fluids, well established by observation for liquids, and extended by analogy to gases." These views involve all the consequences of equality of *vis viva*, in chemical as well as physical actions and reactions, and they indicate the direction in which we may still look hopefully for an extension of our knowledge of chemical physics.

107. *Kinetic and Static Energies.*

Motions, or tendencies to motion, v_1, in elliptical orbits, vary in the inverse ratio of the distance from the centre of gravity. The acceleration of a particle by any given mass, g, varies inversely as the square of the distance. Orbital velocity of a particle, v_0, varies inversely as the square root of the distance. Velocity of gaseous diffusion, v_2, varies inversely as the square root of the density, or inversely as orbital time, or inversely as the $\frac{3}{2}$ power of the mean distance.

$$g \propto \frac{1}{r^2} \propto v_0^4$$

$$v_2 \propto \left(\frac{1}{r}\right)^{\frac{3}{2}} \propto v_0^3$$

$$v_1 \propto \frac{1}{r} \propto v_0^2$$

$$v_0 \propto \left(\frac{1}{r}\right)^{\frac{1}{2}} \propto v_0$$

These several relations all seem likely to be involved in different problems of chemical physics.

108. *Critical Temperatures.*

Notes 58, 92–7, 102–3, indicate a variety of thermodynamic relations to mass, which may be special instances of a large class. Circular, parabolic, and dissociative velocities introduce the factors $\sqrt{2}$ and π, and temperatures vary as the square roots of their representative velocities. Hence may arise an indefinite number of critical temperatures. The two which immediately follow Earth's theoretical internal temperature, Note 103, are 9582°.4 and 12009°.8 F. These temperatures may, perhaps, have important bearings upon questions of specific heat, and specific and atomic *vis viva.*

109. *Harmonic Spectrum of Arsenic* [*and Thallium*].

The American Journal of Science, for September, 1881, publishes Huntington's Arsenic Spectrum, printing in heavy type "the bands which are most brilliant and give character to the spectrum. The other lines are less constant and less distinct, and in some instances may be due to accidental causes." "Upon examining the spectrum it appeared evident that thallium must be present in the arsenic in large quantities." The relations of the observed lines to lines which are in harmonic progression are shown in the following table.

Wave-Length.		Divisors.	
Harmonic.	Observed.	Harmonic.	Observed.
6023	6023		
6012	6013	236x = 1.0018	1.0017
5815	5813	244x 1.0358	1.0361
5653	5653	251x 1.0655	1.0653
5564	5563	255x 1.0825	1.0827
5499	5498	258x 1.0952	1.0955
5334	5340Tl	266x 1.1292	1.1279
5104	5103	278x 1.1801	1.1803
4622	4623	307x 1.3032	1.3028
4592	4593	309x 1.3117	1.3113

The thallium line, 5340, is the only one which differs enough from the harmonic length to throw doubt on its subjection to harmonic influence. If this line were omitted altogether in the calculation, the only change would be in the fourth harmonic divisor, which would be 1.0824 instead of 1.0825. The harmonic wave-lengths would all remain the same as above given.

110. *Mechanical Analogies.*

The researches of Challis and Norton have shown how extensively the various operations of energy may be represented and anticipated by applying the laws of fluid motion. My own kinetic investigations have been greatly helped by my apparatus,* for imitating the "lines of force" which are represented by atmospheric, thermal and electric currents. The telephone, phonograph and photophone furnish practical illustrations of the "mechanical polarization" for which my apparatus was devised. M. C. A. Bjerknes has lately sent a communication to the French Academy "*sur l'imitation, par la voie hydrodynamique, des actions électriques et magnetiques.*" As the indications of unity of force increase, there will be an increased call for the study of such analogies with the help of mechanical contrivances.

111. *Subsident and Parametral Nodes.*

If we take Sun's semi-diameter, Laplace's limit and Earth's nascent locus† as the elements of the stellar-solar paraboloid, we find

$$\eta^2 = 2\,p\,\xi = .9958534$$
$$\xi = .1666667$$
$$2\,p = 5.950342$$

The product of the parameter by any paraboloidal abscissa represents the corresponding ordinal *vis viva*, and the laws of harmonic undulation in elastic media lead us to look for nodes in accordance with the living force. We find numerous evidences of such accordance in the primitive cometoid nebula, if we suppose the nucleal tendencies in the axis of abscissas to be

* For description see Proc. Amer. Phil. Soc., x, 151–66.

† Note 46, *ante*, xix, 446.

arranged in the following order; Neptune (N), Jupiter (J), Mars (M), Sun (S), Mercury (Me), Venus (V), Earth (E), Saturn (Sa) Uranus (U). Let subscript 1, 2, 3, 4, 5 designate, respectively, secular perihelion, mean perihelion, mean, mean aphelion, secular aphelion. Then

1. Subsidence from N_3 would produce a rupturing node at U_4.* Stockwell's values are, $N_3 = 30.034$, $U_4 = 20.044$, the rupturing locus being $\frac{2}{3}$ of $30.034 = 20.023$. Newcomb's estimate of N_3 is 30.070, which would give 20.047 for the rupturing locus.

2. Midway between these primitive nodes comes the primitive nebular centre, J_2. Three of Jupiter's cardinal loci are thus approximately indicated, viz.:

$$(N_3 - U_4) \div 2 = 4.995\ ;\ J_2 = 4.978$$
$$(N_5 - U_4) \div 2 = 5.213\ ;\ J_3 = 5.203$$
$$(N_5 - U_3) \div 2 = 5.425\ ;\ J_4 = 5.427.$$

3. The rupturing locus of the early nebular centre, J_1, and the locus of incipient subsidence at the centre of the belt of greatest condensation, E_5, present the most significant evidence of parametral influence.

$$J_1 + E_5 = 5.954\ ;\ 2p\ E_3 = 5.950$$

4. Other important loci in the dense belt show a like influence.

$$J_3 + V_4 = 5.952$$
$$J_5 + Me_4 = 5.975$$

5. The influence of the parameter on other fundamental abscissas is equally evident.

$$2p\ Me_1 = 1.770\ ;\ M_5 = 1.736$$
$$2p\ M_3 = 9.066\ ;\ Sa_2 = 9.078$$
$$2p\ M_5 = 10.333\ ;\ Sa_5 = 10.343$$
$$2p\ J_2 = 29.622\ ;\ N_1 = 29.598$$
$$2p\ Sa_4 = 59.504\ ;\ 2N_2 = 59.465$$

6. Simple ordinal influence is shown in the relations of Laplace's limit (L) to the centre of condensation, and of the three inner to the three outer planets, if we take Sun's semi-diameter (r_0) as the unit of measurement.

$$\sqrt{2p\ E_4 \div r_0} = 36.322\ ;\ L = 36.366 r_0$$
$$\sqrt{2p\ J_4 \div r_0} = 83.238\ ;\ Me_3 = 83.048 r_0$$
$$\sqrt{2p\ U_3 \div r_0} = 156.491\ ;\ V_3 = 155.184 r_0$$
$$\sqrt{2p\ N_5 \div r_0} = 197.223\ ;\ E_1 = 200.008 r_0$$

7. A similar influence is shown with E_3 as the fundamental abscissa.

$$\sqrt{2p}\ E_3 = 2.439\ ;\ E_4 + M_2 = 2.437$$
$$2\sqrt{2p}\ E_3 = 4.879\ ;\ J_1 = 4.886$$

8. Some of the relative nodes show simple multiples of the parameter.

$$4p = 11.901\ ;\ N_1 - U_1 = 11.910\ ;\ U_5 - Sa_1 = 11.945$$
$$6p = 17.851\ ;\ U_1 = 17.688$$
$$10p = 29.750\ ;\ N_2 = 29.732$$

* Ibid, xvii, 100.

112. *Motion of the Solar System.*

If the interstellar paraboloid (Note 46) were at rest, we should have $\eta = 2\,\xi$; it is, however, equivalent to 5.975ξ. I can think of no other reason for this increase than the absolute motion of the system in space. If the space traversed in a quarter of a solar rotation (6.377 days) is similarly increased we have

$$\xi : \eta\zeta : : r_0 : y$$
$$\text{or } \tfrac{1}{6} : 1.00877 : : r_0 : 6.05264 r_0$$

This would give, for the space traversed in a year, $\frac{365.256}{6.377} \times 6.05264 r_0$. Dividing by 214.45, we obtain 1.6166 times Earth's semi-axis major. Herschel's estimate of the annual motion* is 1.623 semi-axes.

113. *Relation between Jupiter's Mass and Distance.*

The original tendency to relative stability in the positions of the two principal masses of our system, on account of the magnitude of Sun and the nebular-centrality of Jupiter, appears to have made Sun's surface the rupturing locus (secular-perihelion), of the centre of gravity of the two bodies. Stockwell gives, for Jupiter's maximum eccentricity, .0608274; secular perihelion is, therefore, .9391726 of the semi-axis major. If we accept Bessel's mass, we find $1047.879 \div .9391726 = 1115.75 r_0$. Dividing by 5.202798, we find, for Earth's semi-axis major, $214.45 r_0$, which is the value adopted in the British Nautical Almanac.

114. *The Central Sun.*

The true culminating abscissa of the stellar-solar paraboloid (Note 46), is $A_{38} = LM \div r_0$. There is a very large *possible* uncertainty in the distance of *α Centauri,* and even if we take the most recent and careful estimates we may set down the probable error as at least ±.04. Moreover, as the theoretical variability of the abscissas is principally due to variability of æthereal density, A_{39} may represent any point in the orbit of *α Centauri* about the centre of its stellar system. The mean R. A. of *α Centauri* is about 217°53′ and its declination 60°21′S; the opposite pole of the paraboloid, or the theoretical direction of the "Central Sun," if our Sun has been projected from *α Centauri,* should therefore be in the constellation Cassiopeia. The direction of Sun's apparent motion among the fixed stars has been variously estimated between R. A. 252°53′ and 261°22′, and between N. Dec. 38°37′, and 14°26′. Struve's result, from an elaborate discussion of the proper motions of 392 stars, was R. A. 261°22′, Dec. 27°36′, for A. D. 1790. This position is 4°51′ north of the great circle which is at right angles with the axis of the Centaurean paraboloid.

* Outlines of Astronomy, Sect. 853.

115. *The Milky Way and Mädler's Hypothesis.*

Sir John Herschel* objects to Mädler's assignment of "the local centre in space, round which the sun and stars revolve" to the group of the Pleiades, "lying as it does no less than 26° out of the plane of the galactic circle, out of which it is almost inconceivable that any *general* circulation can take place." No such objection can be urged against the radius-vector of the Centaurean paraboloid, for the Milky Way divides at α *Centauri*,* and it traverses Cassiopeia, "its brightest part passing about two degrees to the north of the star δ of that constellation."*

116. *Daily Variations of the Magnetic Needle.*

Sabine's discussions of the magnetic observations at various stations have shown:

1. That the diurnal variation of declination which is due to the Moon's action consists of four equal or nearly equal portions, in which the magnet is attracted alternately to the east and to the west of its mean position.

2. That there is a striking correspondence between the lunar-daily variations of horizontal force and of declination.

3. That in the normal variations of vertical force, the lunar day is also divided into four alternating periods of nearly equal duration, in two of which the force is increased and in the other two it is diminished by the lunar influence.

4. That the lunar-daily variations of inclination and of total force also constitute double progressions, having two maxima and two minima, with alternate periods of increase and decrease, each period being of about six hours' duration.

5. That the solar-daily variations normally constitute only single progressions, of about twelve hours each, from maximum to minimum and from minimum to maximum.

6. That there are, however, "nocturnal episodes" of "retrogressive motion" at some stations, with tendencies to a triple instead of double division of the solar day.

I have shown, by experiment:

7. That any elongated body, when exposed to the action of parallel rectilinear undulations, tends to place itself in the line of those undulations.

8. That this tendency may be increased by giving the elongated body a slight specific energy of direction. For example: if the gimbals of a binnacle compass are so held as to allow motion only in one direction, and the box is made to swing on its free axis like a pendulum, the needle will tend towards the line of oscillation.

The single progression of the solar disturbances (5) and the double progression of the lunar disturbances (1—4) indicate a difference in the character of the chief disturbing influences of the two bodies. It is difficult to

* Op. cit. Sect., 861, 789, 787.

account for such difference by any theory of magnetic induction.* It is evident, however, that Sun's thermal disturbance of Earth's æthereal currents is greater than Moon's, while Moon's tidal disturbance of the same currents is greater than Sun's.

117. *Magnetic Variations at St. Helena.*

St. Helena, on account of its insular position, its proximity to the magnetic equator, its large horizontal force and the large proportion of total force which is represented by the horizontal force, is free from many of the local complications which often mask the normal action of the Sun and Moon. Moreover, the comparatively long period of systematic observations, the extension of the observations to the lunar action on the barometer as well as on the magnetic needle, the uniformity of the indications in different years and in different semesters, and the symmetry which is traceable between the lunar atmospheric and magnetic disturbances, are additional reasons for regarding it as a typical station for the study of gravitating influence on æthereal waves.

118. *Comparative Table.*

The difference between the solar and lunar disturbances is shown in the following synoptical table. The magnetic data are taken from the second volume of the St. Helena Observations, pp. xlii-xliv and lviii-lxii; the barometric, from the first volume, pp. 84, 99. The barometric *ratios* are given, in order to make the table homogeneous and facilitate comparison. They were obtained by dividing the hourly deviations from the mean by the mean height of the barometer (28.278 in.).

Solar Disturbances.

h.	Bar.	V. F.	H. F.	T. F.
⊙	.00	.000	.00	.000
0	+0566	—022	+1099	+95
1	—0035	+229	+0911	+82
2	—0530	+446	+0623	+60
3	—0954	+593	+0368	+40
4	—1061	+638	+0133	+20
5	—0920	+608	—0080	+01
6	—0636	+611	—0270	—15
7	—0212	+545	—0394	—26
8	+0247	+300	—0465	—36
9	+0636	+219	—0511	—41
10	+0848	+074	—0530	—45
11	+0742	—011	—0522	—45
12	+0354	—100	—0481	—43
13	—0106	—165	—0449	—41

*See paper by Dr. Lloyd, Proc. R. Irish Acad., 1858, and C. Chambers, Phil. Trans., 1863.

Solar Disturbances.

h.	Bar.	V. F.	H. F.	T. F.
14	—0566	—224	—0405	—38
15	—0884	—289	—0376	—36
16	—0919	—345	—0352	—35
17	—0742	—398	—0329	—34
18	—0247	—465	—0298	—32
19	+0318	—513	—0154	—20
20	+0813	—582	+0130	+03
21	+1202	—491	+0470	+34
22	+1308	—425	+0803	+63
23	+1061	—214	+1019	+85

Lunar Disturbances.

h.	Bar.	V. F.	H. F.	T. F.
☽	.000	.0000	.0000	.0000
0	+619	—05	+06	+05
1	+523	+27	—03	—01
2	+300	+31	—11	—05
3	—035	+44	—14	—06
4	—318	+72	—20	—07
5	—477	+41	—14	—06
6	—749	+50	—07	+01
7	—619	+28	—04	+01
8	—290	—12	00	—02
9	+007	—11	+22	+18
10	+205	—17	+32	+25
11	+488	—37	+31	+22
12	+566	—03	+19	+16
13	+417	+05	+17	+15
14	+194	+19	+13	+14
15	—255	+48	—09	—01
16	—417	+51	—11	—02
17	—530	+29	—08	—03
18	—644	+13	—09	—06
19	—513	—11	—04	—05
20	—290	—53	+03	—05
21	—265	—50	+06	—02
22	+407	—54	+13	+04
23	+548	—67	+09	—01

119. *Magnetic Correlations.*

In studying the above table for a proper interpretation of its indications, we can follow no better clew than the known correlations of electricity and magnetism, which may be classed under the following heads :

a. Friction.—Thermal and tidal currents, combined with the resistance of the Earth's surface, must produce atmospheric friction.

β. Chemical Action.—The evidences of incessant dissociation and re-association in the solar photosphere, are so conclusive, that chemical action may be very properly regarded as an important source of specific solar magnetism.

γ. Light.—The numerical equivalences between various forms of luminous, gravitating, and electro-magnetic action, are so striking as to justify Maxwell in the assertion that "the properties of the electro-magnetic medium are identical with those of the luminiferous medium."*

δ. Heat.—Edlund has shown that many of the phenomena of heat and electricity may be explained by the hypothesis of two forms of motion in the same elastic æthereal medium.

ε. Rotation.—Arago, Babbage, Herschel, Barlow, Christie, Chase, and Perry and Ayrton, have shown that simple rotation produces magnetical disturbances which are governed by fixed laws.

ζ. Gravitation.—In addition to the relation which I have shown to exist between solar rotation and luminous velocity, it is evident that electricity must be modified by pressure and by such changes in the relative distances of electrified particles as are produced by disturbances of gravitating equilibrium.

η. Currents.—A comparison of oceanic currents with the cotidal lines, the lines of isabnormal temperature and the magnetic lines of equal declination, shows such points of resemblance as to make it probable that they are all due to the action of the same forces upon different media, or under different circumstances. Challis has found that if all the ordinary central forces are due to transformed æthereal vibrations, "the actions of such forces on atoms are in every instance attributable to *æthereal currents*, whether the atoms be *immediately* acted upon by steady motions of the æther or by æthereal vibrations." †

120. *Comparison of Solar Daily Magnetic and Meteorological Means.*

The solar-daily maximum of vertical force at St. Helena is coincident with the daily barometric minimum, as well as with the minimum of gaseous pressure and of mean pressure of the wind. The daily minimum of vertical force (20 h) is approximately coincident with the daily maximum of the barometer (22 h), of gaseous pressure (21 h), and of wind pressure (22–23 h). The most rapid increase of vertical force is between 22 h and 3 h, when the barometer is falling and the gaseous pressure diminishing; the most rapid diminution is between 7 h and 10 h, when the barometer is rising and the gaseous pressure increasing. The range of total force between noon and midnight (.00095 + .00043 = .00138), is of the same order of magnitude as the daily range of solar disturbances of weight or pressure $\left(\frac{m}{d^2_3} = .000615\right.$, which is added at midnight and subtracted at

* *Electricity and Magnetism*, ii, 383, sqq.

† *Phil. Mag.*, Sept. 1872; Sept. 1876; June, 1878.

noon, making the total range .00123). The arithmetical mean of any cumulative disturbances which are occasioned by the accelerations and retardations of alternate half-daily fall towards and rise from the Sun, should occur at 12h $\div \sqrt{2}$ from midnight, or at 8h. 29m. A.M. and 3h. 31m. P.M. These hours correspond very nearly with those of mean disturbance, both in the horizontal and in the total force :

	Noon.	12 P.M.	Mean.	Time of Mean.
H. F.	+.001099	—.000481	+.000309	8h. 31m. A.M.; 3h. 16m. P.M.
T. F.	+.00095	—.00043	+.00026	8h. 45m. A.M.; 3h. 42m. P.M.

The greatest observed deviation from the theoretical times is 16m, in the morning mean of total force ; the least, 2m., in the morning mean of horizontal force. The mean deviation of horizontal force is 8.5m.; of total force, 2.5m.

121. *Lunar Daily Comparison.*

The lunar disturbances, both of the barometric and of the magnetic means, are of a higher order than can be accounted for by mere disturbances of weight or pressure, or by any other known activity of our satellite except the accumulation of energy in currents (η). We have no reason to think that the moon exerts any specific chemical (β), luminous (γ), or thermal (δ) influence of her own, but her tidal disturbances of the elastic or *quasi*-elastic currents of the rotating earth (ε, ζ, η) are very important.

Schiapparelli, Loomis and Chase have shown that long-continued observations, at various stations, demonstrate the existence of an evident lunar influence on the precipitation of rain, and, consequently, on the electrical condition of the atmosphere. Each station has an "establishment" of its own, which can be determined, where the meteorological conditions are most uniform, by observations extending over a period of three or four years. This influence, which is undoubtedly due to tidal disturbances of atmospheric currents, is further shown in the lunar modifications of the direction and velocity of the wind, which have been pointed out by M. Bouquet de la Grye.*

Both in the lunar and in the solar tables the critical periods of horizontal and total force are nearly synchronous. In the lunar variations the vertical force increases as the horizontal force diminishes, and *vice-versâ.* Each of the lunar magnetic tides is of the same general character as the oceanic and atmospheric tides. The lunar horizontal force and total force are greatest and the vertical force is least when the barometric currents are moving most rapidly away from the Earth's centre ; the horizontal and total force are least and the vertical force is greatest when the currents are moving most rapidly towards the centre.

The range of lunar disturbances of vertical force (from + .000072 to

* *Comptes Rendus*, lxxxviii, 345-8.

— .000067 = .000139) is almost identical with the range of lunar barometric disturbances (from — .000075 to + .000062 = .000137).

The culminations of the lunar disturbances, both of the vertical and of the horizontal force, correspond approximately with the mean sum of the accelerations and retardations of lunar tidal action by terrestrial rotation ($6h \div \sqrt{2} = 4h\ 14.6m$ after 0h, 6h, 12h, 18h.)

Vertical Force.		Horizontal Force.	
1st Max.	0h + 3h 57m	1st Min.	0h. + 4h 1.3m
1st Min.	6h + 4h 51.5m	1st Max.	6h. + 4h 24.4m
2d Max.	12h + 3h 36.1m	2d Min.	12h. + 3h 44m
2d Min.	18h + 4h 39.8m	2d Max.	18h. + 4h 9.2m
Mean	4h 16.1m	Mean	4h 4.7m

122. *Conclusions.*

Although the barometric observations furnish the most ready data for quantitative measurements and comparisons, the combined action of terrestrial rotation with lunar tidal and terrestrial equilibrating gravitation is not confined to the air. Every particle of the globe is continually subject to cyclical variations of stress and strain. In the first and third quadrants the lunar action is opposed, while in the second and fourth it is aided, by terrestrial rotation, so that the resultant of all the subterranean magnetic influences must be subject to lunar disturbances of the same character as those which modify the barometric and electric currents in the atmosphere.

We may, therefore, conclude that the solar disturbance of the terrestrial magnetic currents is chiefly and primarily due to its thermal activity ; the lunar, to gravitating currents which are modified by terrestrial rotation and orbital revolution.

123. *"Forced Oscillation."*

In discussing the synchronism of the motion of the moon's nodes with terrestrial nutation Herschel* introduces "the principle of forced oscillations, or of forced vibrations," by the following announcement :

"If one part of any system connected either by material ties, or by the mutual attractions of its members, be continually maintained by any cause, whether inherent in the constitution of the system or external to it, in a state of regular periodic motion, that motion will be propagated throughout the whole system, and will give rise, in every member of it and in every part of each member, to periodic movements executed in equal period with that to which they owe their origin, though not necessarily synchronous with them in their maxima and minima."

A demonstration of this theorem for the forced vibrations of systems connected by material ties of imperfect elasticity, is given in Herschel's Treatise on Sound.† Fourier's theorem, Herschel's theory of the consequences of

* *Op. cit.*, Sect. 650.

† Encyc. Metrop., Art. 323.

nebular subsidence, and the various forms of harmonic synchronism are all dependent upon the same fundamental principles and they should all be kept in mind by those who are investigating the consequences of elastic action and reaction.

124. *Fundamental Photodynamic Nodes.*

The principle of forced vibrations, the theory of subsidence, and the laws of varying density in elastic media, are illustrated by the equation,

$$\left(\frac{L}{r}\right)^{\frac{L}{r}} = \left(\frac{M}{r}\right)^{\frac{i}{c}}$$

$$\therefore \log. L : \log. M :: \frac{i}{c} : \frac{L}{r}$$

$L =$ Laplace's limit of synchronous solar rotation and revolution ; $M =$ modulus of light at Sun's surface ; $r =$ Sun's semi-diameter ; $i =$ locus of mean incipient subsidence for the nebular centre of planetary inertia, (Saturn's mean aphelion); $c =$ central locus of greatest belt-condensation (Earth's semi-axis major).

If we adopt the values for L and M which are given in Note 46, we find $i = 9.99861\,c$. Stockwell's value is about $\frac{1}{70}$ of one per cent. greater, or $10.000059\,c$.

125. *A "Derivative Oscillation."*

After announcing the principle of forced vibrations, Herschel continues as follows : "The system may be favorably or unfavorably constituted for such a transfer of periodic movements, or favorably in some of its parts and unfavorably in others ; and accordingly as it is the one or the other, the *derivative* oscillation (as it may be termed) will be imperceptible in one case, of appreciable magnitude in another, and even more perceptible in its visible effects than the original cause in a third ; of this kind we have an instance in the Moon's acceleration."

A harmonic illustration of this statement is furnished by the lunar disturbance of vertical magnetic force at St. Helena. Von Littrow's estimate of Moon's semi-axis major is 60.2778. Earth's action upon Moon and Moon's reaction should therefore be nearly $\frac{1}{3633}$ as great as its action at its own surface. If the resulting waves or "forced vibrations" are reflected to Earth and resolved, one-half into vertical force and one-half into horizontal force, the consequent disturbance should be $\frac{1}{7267}$. The lunar disturbance of vertical force (Note 121) is $\frac{1}{7194}$; of barometer, $\frac{1}{7299}$. The decimal values are :

Disturbance of vertical force		.0000139
"	" vibrations	.0000138
"	" barometer	.0000137

The derivative oscillation in the horizontal force is obscured by other disturbances.

126. *Relation of Magnetic Disturbances to Thermal Currents.*

The secondary character of the solar disturbances of vertical force at St. Helena is further indicated by a comparison of culminating times. The coincidence of the daily minimum of vertical force with the various maxima of gravitating pressures (Note 120) is only approximate; but the relation both of maximum and of minimum to the culmination of ascending and descending currents is very close. The mean time of maximum temperature is 1 h 39.5 m; of minimum temperature, 17 h 41.6 m; of greatest vertical force, 4 h 4.4 m; of least vertical force, 19 h 55.8 m. Therefore, the vertical force continues to increase for 2 h 24.9 m after the time of greatest daily heat, and to diminish for 2 h 14.2 m after the time of greatest daily cold.

127. *Relation of Barometric to Gravitating Disturbances.*

During the forced vibrations of half-daily terrestrial rotation towards and from the Sun, terrestrial gravity acts on all the heated and otherwise disturbed particles of the atmosphere. The sum of the cyclical accelerations, at St. Helena, is $\frac{16.05 \times 43082^2}{5280} = 5642000$ miles $= a$. The sum of the synchronous "forced oscillations" of rotation towards or from the Sun is cos. $15^\circ\, 56'\, 41'' \times \pi \times 3962.8 = 11970.7$ miles $= \beta$. The mean sum of the barometric pressures of the atmospheric particles is 28.278 inches $= \gamma$. The sum of the mean half-daily disturbances of pressure is $\frac{1}{2}$ $(28.315 - 28.248 + 28.302 - 28.252) = .0585$ inch $= \delta$. The ratio of α to β is nearly the same as that of γ to δ; $\frac{\beta}{a} = .002122$; $\frac{\delta}{\gamma} = .002069$; the deviation from exact accordance being about 2.56 per cent.

128. *Succession of Forced Oscillations.*

The "nascent" velocity of the Sun, or the limiting velocity between complete dissociation and incipient aggregation, is, as we have seen, $\frac{g_0 t_0}{2}$ = velocity of light. On the principle of forced oscillations, the luminiferous æthereal undulations force the sun into rotary oscillations synchronous with the cycles of superficial gravitating activity which would communicate the velocity of light.

The ratio of the mass aggregation at the principal centre of nucleation (Sun), to the mass aggregation at primitive nebular centre (Jupiter), is, as we have seen, the same as the ratio of the rupturing radius-vector of Jupiter to the rupturing radius-vector of Sun. The rupturing energy of the æthereal oscillations is thus traceable to the primitive condition of statical equilibrium, when the two products of mass by rupturing radius were equal.

These two principal masses of the solar system tend to produce a system of forced oscillations through their common centre of gravity. Circular orbital velocity at their mean centre of gravity has become "nascent" velocity for the mass at the centre of the belt of greatest condensation (Earth).

129. *Radius of Primitive Condensation.*

The importance of the three masses, which were considered in the foregoing note, seems to strengthen the probability that the resulting oscillations are *exactly* recorded in relations of mass, distance and velocity. If such is the case, Earth's semi-axis major may be easily computed.

1. Earth's nascent velocity is $\frac{g_3 t_3}{2} = \frac{32.087 \times 43082}{5280} = 261.81$ miles.

2. The mean locus of the centre of gravity of Sun and Jupiter is $\frac{5.2028}{1047.879 + 1}$ of Earth's semi-axis major.

3. If $\sqrt{g_0 r_0} = 261.81$, Earth's orbital velocity is $\left(\frac{5.2028}{1047.879 + 1}\right)^{\frac{1}{2}} \times$ $261.81 = 18.4392$ miles per second.

4. This would give, for Earth's semi-axis major, 18.4392×1 year in seconds $\div 2\pi = 92,613,000$ miles.

130. *Paraboloidal Inclination.*

The progression of the stellar-paraboloidal abscissas may be expressed under the form $\xi(\eta\zeta^n)^n$, positive values of n giving centripetal, and negative values giving centrifugal abscissas. The η ordinates are, therefore, modified by ζ, as often as the fundamental abscissa is modified by the *modified* ordinate. The ζ modification seems to imply a cyclical elliptic-influence consequent upon rotation. If ζ is the semi-axis major and η the semi-axis minor of the ellipse, the inclination of the ellipse to the η circle may be readily found by the equation

$$\text{Sec. } \varphi = \frac{\zeta}{\eta} = \frac{1.012974}{.9958534}$$

$$\varphi = 10^\circ\ 32'\ 56''.$$

131. *Other Cometary Hypotheses.*

The hypothesis in Note 114 is only one out of many which might be assumed with nearly equal probability. The great FACT which is to be accounted for is the evidence of paraboloidal influence, extending from the region of the nearest fixed stars to our Sun, and affecting intra-nucleal nodal condensations, revolutions and rotations, through cyclical undulations which are harmonically determined by the inertia of Sun's mass and the velocity of light. I have already shown some of the important modifications which are introduced into the nebular hypothesis by these evidences of paraboloidal or cometoid subsidence, and I look confidently for the discovery of many others. The connection of comets with meteoric streams will, doubtless, awaken many inquiries respecting stellar groups and stellar motions, some of which may be within the reach and others beyond the reach of future satisfactory solution.

132. *Further Paraboloidal Harmony.*

Let r_a be a fourth proportional to Jupiter's locus of incipient subsidence,* Earth's semi-axis major and Sun's semi-diameter. Then the first three abscissas of Note 46 ($A_0 — A_2$) determine the paraboloid, and there are four groups of 3^2 abscissas, between r_a and the fixed stars, viz.: 9, ($A_3 — A_{11}$), between r_a and Sun's semi-diameter; 9, ($A_{12} — A_{20}$), between Sun's surface and the loci of planetary rupture; 9, ($A_{21} — A_{29}$), within the belt of rupturing loci; 9, ($A_{30} — A_{38}$), between the rupturing loci and *α Centauri*. The influence near the centre of the belt of greatest condensation appears to be also traceable in the ratio between the rupturing abscissas for Earth and Venus, $A_{23} \div A_{22} = 1.77875$,

$$\begin{aligned}
\log. 1.77875^{-3} &= -.7503432 \\
\log. 1.77875^{0} &= .0000000 \\
\log. 1.77875^{6} &= 1.5006864 \\
\log. 1.77875^{16} &= 4.0018304 \\
\log. 1.77875^{30} &= 7.5034320
\end{aligned}$$

By referring to the table in Note 46, it will be seen that the division of the abscissas into four groups of nine each is indicated by these logarithms. It may be well also to observe that 1.77875^4 differs by less than $\frac{1}{10}$ of one per cent. from Stockwell's value for Saturn's mean aphelion, estimated in terms of Earth's semi-axis major.

133. *Another Confirmation of Prediction.*

On the 4th of October, 1878, I presented a communication to the American Philosophical Society,† in which I showed that the position of Watson's first intra-Mercurial planet, as computed by Gaillot and Mouchez, represented the third intra-Mercurial term of my harmonic series. At the last meeting of the British Association, Prof. Balfour Stewart read a paper in which he gave indications of sun-spot disturbances by a planet, revolving in 24.011 days, and consequently having a semi-axis major of .163. This confirmation, both of my own prediction‡ and of the calculations of the French astronomers, is the more interesting, because the first confirmation of my series was contained in a communication which was made to the Royal Society by Messrs. De la Rue, Stewart and Loewy, forty-one days

*Secular aphelion.

†Proc. Am. Phil. Soc., xviii, 34-6.

‡*Ib.* xiii, 238.

after I had announced the series to the Philosophical Society and published it in the New York Tribune.* The accordances are as follows :

Prediction.		Confirmation.	
1st interior harmonic term....	.267	De La Rue, S. and L........	.267
3d " " "	.165	Gaillot and Mouchez........	.164
		Stewart....................	.163

134. *Prout's Hypothesis.*

Clarke (P. Mag. [5] xii, 109-10), gives the results of his re-calculation of atomic weights, which inclines him to look favorably on Prout's hypothesis, although he had previously believed that it had been forever overthrown. Maximilian Gerber (Les Mondes ; cited in Chemical News, xliii, 242-3), rejects the hypothesis, but he gives four additional empirical units, which seem to indicate a probability that groups of similar valency may have special common divisors. The varied evidences of the photodynamic importance of hydrogen will doubtless incline chemists to give weighty consideration to Clarke's deliberate opinion, and Gerber's factors may help towards its establishment. The possibility of measuring undulatory *vis viva* by the distance of projection against uniform resistance, as well as by orbital areas, may, perhaps, furnish the requisite clue for reconciling apparent oppositions of indication.

135. *Phyllotactic Atomicity.*

The phyllotactic law distributes leaves and branches evenly around the stems of vegetables, so that all parts of the plant may share in the benefit of heat, air and moisture. In 1849, Dr. Thomas Hill, at the request of Prof. Peirce, showed that the times of planetary revolution are phyllotactic, and the planets are thus distributed around the Sun so evenly as to avoid the destruction of the system by the accumulated perturbations of the great planets.† If the several atomic elements have special systems of æthereal vibrations, it seems reasonable to look for evidences of a phyllotactic harmony which would contribute to the stability of equilibrium in compounds. The following table, which includes about half the known elements, contains multiples of the phyllotactic divisor $\frac{8}{5}$ H, or 1.6 H, compared with Clarke's recalculation of atomic weights.

	Phyllotactic.	Clarke.	Difference.
O	$10 \times 1.6 = 16$	15.963	.037
Fl	$12 \times 1.6 = 19.2$	18.984	.216
Mg	$15 \times 1.6 = 24$	23.951	.049
S	$20 \times 1.6 = 32$	31.984	.016
Cl	$22 \times 1.6 = 35.2$	35.370	.170
Ca	$25 \times 1.6 = 40$	39.990	.010
Ti	$31 \times 1.6 = 49.6$	49.846	.246

* *Ib.* p. 470.

† Proc. Amer. Assoc., vol. 2.

	Phyllotactic.	Clarke.	Difference.
V	32 × 1.6 = 51.2	51.256	.056
Se	49 × 1.6 = 78.4	78.797	.397
Br	50 × 1.6 = 80	79.768	.032
Zr	56 × 1.6 = 89.6	89.367	.233
I	79 × 1.6 = 126.4	126.557	.157
Te	80 × 1.6 = 128	127.960	.040
Cs	83 × 1.6 = 132.8	132.583	.217
Ytter	108 × 1.6 = 172.8	172.761	.039
Bo	7 × 1.6 = 11.2	10.940	.260
Al	17 × 1.6 = 27.2	27.009	.191
Fe	35 × 1.6 = 56	55.913	.087
Ga	43 × 1.6 = 68.8	68.854	.054
Cd	70 × 1.6 = 112	111.770	.230
In	71 × 1.6 = 113.6	113.398	.202
Yt	56 × 1.6 = 89.6	89.816	.216
Ru	65 × 1.6 = 104	104.217	.217
Ro	65 × 1.6 = 104	104.055	.055
Rh	66 × 1.6 = 105.6	105.737	.137
Sb	75 × 1.6 = 120	119.955	.045
Ta	114 × 1.6 = 182.4	182.144	.256
W	115 × 1.6 = 184	183.610	.390
Os	124 × 1.6 = 198.4	198.494	.094
Hg	125 × 1.6 = 200	199.712	.288
Pb	129 × 1.6 = 206.4	206.471	.071
Th	146 × 1.6 = 233.6	233.414	.186
U	149 × 1.6 = 238.4	238.482	.082

The greatest difference in the above table of 33 elements is less than 25 per cent. of the phyllotactic unit. Gerber's longest table, for a single divisor, contains but 25 elements; his greatest difference is more than 36 per cent. of his empirical divisor. If the eight elements which present the greatest phyllotactic difference were rejected, so as to make the tables of the same length, the greatest remaining difference would be less than 15 per cent. of the divisor. The photodynamic approximation is therefore much closer than the empirical.

136. *The Empirical Divisors are Phyllotactic.*

Gerber says that "no simple relation exists among" his divisors, therefore they "have no value in themselves." The relation which he was unable to discover, is phyllotactic, as will be seen by the following comparisons:

Gerber.		Phyllotactic.	
H	.9997	H	.998
D_1	.769	$\frac{5}{13} \times 2$ H	.768
D_2	1.995	2 H	1.996
D_3	1.559	$\frac{5}{2} \times \frac{5}{8}$ H	1.559
D_4	1.245	$\frac{5}{2} \times \frac{1}{2}$ H	1.247

The first six phyllotactic numbers are 1, 2, 3, 5, 8, 13; the third does not appear in the formation of the theoretical divisors, but the others are all employed. The simple phyllotactic relation of all the divisors to H, shows that they *have* "value in themselves."

137. *Mutual Phyllotaxy.*

Upon examining the mutual relations of the above phyllotactic divisors, it will be seen that $D_1 = \frac{5}{13} D_2 = 2 \times \frac{2}{5} \times \frac{8}{13} D_3 = \frac{8}{13} D_4$; $D_2 = 2 \times \frac{2}{5} \times \frac{8}{5} D_3 = 2^2 \times \frac{2}{5} D_4$; $D_3 = \frac{5}{2} \times \frac{1}{2} D_4$; $D_4 = \sqrt{H\, D_3}$. These varied provisions for the stability of cyclical equilibrium, in all possible varieties of intra-molecular æthereal movement, show that the command, "Let there be light," manifested its formative power of organization as soon as material atoms were set in motion. The appearance of the first five phyllotactic numbers, 1, 2, 3, 5, 8, in crystallization, furnishes a step from inorganic to organic morphology, giving new meaning to the landscapes on our frosted window-panes, as well as to the protective mimicry of vegetables and animals, as illustrations of the "distributive ratio" which controls alike light-waves, atomic inertia, crystalline structure, organic growth, planetary configuration and interstellar action.

138. *Relations of the Water Molecules.*

The importance of oxygen and hydrogen, both in mutual combination and in connection with other elements, suggests the following comparative grouping of Clarke's table of atomic weights:

O = 16;	H = 1.0023.	Difference.	O = 16;	H = 1.0023.	Difference.
Br	79.951	.049	Bi	208.001	.001
I	126.848	.152	Pb	206.946	.054
Mg	24.014	.014	Mn	54.029	.029
Zn	65.054	.054	Fe	56.042	.042
Cs	132.918	.082	Ni	58.062	.062
Ag	107.923	.077	Co	59.023	.023
Tl	204.183	.183	Bo	10.966	.034
Se	78.978	.022	Ga	68.963	.037
Mo	95.747	.253	Ce	140.747	.253
W	184.032	.032	Yttr	90 023	.023
U	239.030	.030	Ytter	173.158	.158
P	31.029	.029	La	138.844	.156
Cd	112.027	.027	Di	144.906	.094
Hg	200.171	.171	Th	233.951	.049
Ba	137.007	.007	Pt	194,867	.133
C	12.001	.001	Ir	193.094	.094
Ti	49.961	.039	Os	198.951	.049
Sn	117.968	.032	Pd	105.981	.019
In	113,659	.341			

H = 1; O = 15.9633.		Difference.	H = 1; O = 15.9633.		Difference.
Fl	18.984	.016	Zr	89.367	.367
Cl	35.370	.370	N	14.021	.021
Li	7.007	.007	Sb	119.955	.045
Gl	9.085	.085	Ta	182.144	.144
Na	22.998	.002	Sc	43.980	.020
K	39.019	.019	Al	27.009	.009
Rb	85.251	.251	V	51.256	.256
S	31.984	.016	As	74.918	.082
Te	127.960	.040	Cu	63.173	.173
Cr	52.009	.009	Er	165.891	.109
Ca	39.990	.010	Rh	104.055	.055
Sr	87.374	.374	Ru	104.217	.217
Si	28.195	.195	Au	196.155	.155

The above tables seem to show that, if Prout's law is correct, the value of the oxygen atom has been more accurately determined than that of the hydrogen atom. The deviations of Mo, In, Ce, Cl, Rb, Sr, Zr, and V, are so great as to require some explanation, which may, perhaps, be found in phyllotactic or harmonic influence.

139. *Further Evidence of Phyllotaxy.*

Upon further examination of Gerber's tables, I find a still closer agreement with rigidly phyllotactic divisors. Taking the mean of the estimated atomic weights, when two are given, his values should be as follows: $D_1 = 655.28 \div 853 = .7681$; $D_2 = 1440.4 \div 722 = 1.9950$; $D_3 = 646.21 \div 415 = 1.5571$; $D_4 = 2763.05 \div 2218 = 1.2457$; $H = 1.3\ D_1 = 9986$. Clarke's values for the atomicities, when grouped in the same way as Gerber's, give $D_1 = 656.212 \div 853 = .7693$; $D_2 = 1437.96 \div 721 = 1.9944$; $D_3 = 643.53 \div 413 = 1.5582$; $D_4 = 2752.051 \div 2208 = 1.2464$; $H = 1.3\ D_1 = 1.0001$. The following table shows the nearness of agreement between the empirical and the phyllotactic divisors:

	Phyllotactic.	Gerber.	Clarke.
$H = \frac{1}{16} O$	.9977	.9986	1.0001
$D_1 = \frac{10}{13} H$	.7675	.7681	.7693
$D_2 = 2\ H$	1.9954	1.9950	1.9944
$D_3 = \frac{5}{4}\ D_4$	1.5589	1.5571	1.5582
$D_4 = \frac{5}{8}\ D_2$	1.2471	1.2457	1.2464

140. "*Elasticity.*"

"Sir W. Thomson is led from the consideration of various experiments with fluids and solids and the study of smoke rings to speculate upon elasticity as an evidence of motion. The kinetic theory of gases requires that the molecule or atom shall be elastic. 'But this kinetic theory of matter is a dream and must remain so until it can explain chemical affinity, electricity, magnetism, gravitation and inertia.' The writer looks forward to a greater generalization which shall include elasticity as a form of motion."—J. T., in *Am. Jour. of Science*, Nov. 1881.

My first physical paper (Proc. Am. Phil. Soc., ix, 283-8), deduced approximate values of solar mass and distance from the combined action of daily rotation, yearly revolution and atmospheric elasticity. All my subsequent radiodynamic investigations have been based upon the consideration of the various forms of harmonic relation which *ought* to follow from the undulations of an all-pervading elastic medium, such as the luminiferous æther is generally supposed to be.

141. *Harmonic Spectra.*

Schuster (*Proc. Roy. Soc.*, xxxi, 337-47), discusses the probability of accidental harmonic coincidences in spectral wave-lengths, giving the following summary of his results for the iron spectrum :—

"1. *There is a real cause acting in a direction opposed to the law of harmonic ratios, so far as fractions formed by numbers smaller than seventy are concerned.*

"2. *After elimination of the first cause a tendency appears for fractions formed by two lines to cluster round harmonic ratios.*

"3. *Most probably some law hitherto undiscovered exists, which in special cases resolves itself into the law of harmonic ratios.*"

The comparison between the planetary harmonic roots and the spectral harmonic quotients (Note 37), suggests the probability that the opposition to strict harmonic ratios may be due to differences of inertia in the wave-systems, which would be more rigidly harmonic were it not for such differences. The simple tendency of all elastic media to harmonic vibrations would then be the general law, instead of a law which becomes harmonic "in special cases."

In waves which are propagated with such rapidity as those of light, it seems reasonable that there may be large factors of harmonic length, which are modified by smaller disturbing elements. Schuster's analysis does not reach such cases as are given in Notes 36, and 39–42. Note 36 gives 11 harmonic divisors, which deviate from the observed divisors by a mean amount of less than $\frac{1}{19}$ of one per cent. The greatest difference is in the C line, where the harmonic divisor is 1.1530, the observed divisor being 1.1592, giving a deviation of $\frac{31}{58}$ of one per cent. In Note 39, the greatest difference between either of the harmonic lines and the corresponding basic line is $\frac{1}{3}$ of one per cent. In Note 41 the greatest difference is $\frac{1}{20}$ of one per cent. In Note 42, $\frac{1}{14}$ of one per cent.

142. *The Magnesium Spectra.*

Liveing and Dewar (Proc. Roy. Soc., xxxii, 189–203), give some results of their investigations on the spectrum of magnesium, which seem to strengthen the probability of large harmonic factors, modified by small disturbances. The only two single lines which are found in the flame-spectrum, the arc-spectrum and the spark-spectrum, have wave-lengths of 2850 and 4570, respectively. These represent, approximately, the phyllotactic numbers 5 and 8, viz.:

Phyllotactic.	Observed.
2854	2850
4566	4570

In the arc-spectrum and spark-spectrum, there is "a very striking group of two very strong lines at wave-lengths about 2801 and 2794," and "one line common to the arc and spark at wave-length 4703" which "does not appear in Angstrom's table." The difference between the "two very strong lines" has modified the other lines, as shown below :—

Harmonic.	Observed.
$7 \times 399 = 2793$	2794
$7 \times 400 = 2800$	2801
$7 \times 407 = 2849$	2850
$7 \times 653 = 4571$	4570
$7 \times 672 = 4704$	4703

The greatest difference is $\frac{1}{7}$ and the mean difference is $\frac{1}{35}$ of the harmonic divisor. If there were no law controlling the approximations, the probable maximum difference would be $\frac{1}{2}$ and the mean difference would be .25 of the divisor.

143. *The Special Phyllotactic Elements.*

The elements which cannot be simply grouped with O and H, within the limits of probable error (Note 138), have the following phyllotactic relations to oxygen,

	Observed.	Phyllotactic.	Difference.
Mo	$95.747 \times \frac{1}{16}$ O	$2^5 \times 3 = 96.$	.253
In	$113.659 \times \frac{1}{16}$ O	$7 \times 13 \times \frac{5}{4} = 113.75$	.091
Ce	$140.747 \times \frac{1}{16}$ O	$3 \times 61 \times \frac{10}{13} = 140.769$	.022
Cl	$35.451 \times \frac{1}{16}$ O	$2 \times 23 \times \frac{10}{13} = 35.385$	.066
Rb	$85.529 \times \frac{1}{16}$ O	$3 \times 37 \times \frac{10}{13} = 85.385$	.144
Sr	$87.575 \times \frac{1}{16}$ O	$5^2 \times 7 \times \frac{1}{2} = 87.5$	.075
Zr	$89.573 \times \frac{1}{16}$ O	$11 \times 13 \times \frac{5}{8} = 89.375$	.198
V	$51.373 \times \frac{1}{16}$ O	$41 \times \frac{5}{4} = 51.25$	.123

144. *Analysis of the Hydrogen Spectrum.*

It seems reasonable to look for clearer evidence of undisturbed or slightly modified harmonic influence in hydrogen, than in any of the heavier elements. Accordingly Professor Johnstone Stoney has "shown that three out of the four lines in the visible part of the spectrum have wave lengths which, to a high degree of accuracy, are in the ratios of 20 : 27 : 32." I find, moreover, that three of the lines are in simple geometric ratio, as will be seen by the following comparison :

Theoretical Harmonic Lines.	Observed.
$\alpha = (2 \times 3)^3 \times 30.379 = 6561.8$	6561.8
$\beta = 5 \times 2^5 \times 30.379 = 4860.6$	4860.6
$\gamma = (\delta^2\,\beta)^{\frac{1}{3}} = 4340.1$	4340.
$\delta = 5 \times 3^3 \times 30.379 = 4101.1$	4101.2

The extreme lines are phyllotactic, δ being $\frac{5}{8}$ of α. The greatest discrepancy is $\frac{1}{410}$ of one per cent. which is unquestionably within the limits of probable error.

145. *Application of Schuster's Tests.*

The ratio between the hydrogen lines α and δ is between $\frac{8}{5}$ and $\frac{3}{2}$.

$$\begin{aligned} \alpha \div \delta &= a = 1.59997 \\ 8 \div 5 &= b = 1.60000 \\ 3 \div 2 &= c = 1.50000 \\ b - c &= d = .10000 \\ b - a &= e = .00003 \end{aligned}$$

$e \div d = \frac{3}{10000}$, or less than $\frac{1}{83}$ of one per cent. of the probable error.

The ratio between γ and δ is between $\frac{91}{86}$ and $\frac{18}{17}$

$$\begin{aligned} \gamma \div \delta &= a_1 = 1.058227 \\ 18 \div 17 &= b_1 = 1.058824 \\ 91 \div 86 &= c_1 = 1.058140 \\ b_1 - c_1 &= d_1 = .000684 \\ a_1 - c_1 &= e_1 = .000087 \end{aligned}$$

$e_1 \div d_1 = \frac{29}{228}$, the probable error being $\pm \frac{57}{228}$.

The ratio between α and β is between $\frac{27}{20}$ and $\frac{77}{57}$.

$$\begin{aligned} \alpha \div \beta &= a_2 = 1.350165 \\ 77 \div 57 &= b_2 = 1.350877 \\ 27 \div 20 &= c_2 = 1.350000 \\ b_2 - c_2 &= d_2 = .000877 \\ a_2 - c_2 &= e_2 = .000165 \end{aligned}$$

$e_2 \div d_2 = \frac{165}{877}$, the probable error being $\pm \frac{219}{877}$. If we were to stop here, the test would be pronounced satisfactory, and the evidence of harmonic influences in which all the lines are involved would be conclusive. But if we try another mode of grouping the result will be different.

The ratio between β and δ is between $\frac{32}{27}$ and $\frac{77}{65}$.

$$\begin{aligned} \beta \div \delta &= a_3 = 1.185009 \\ 77 \div 65 &= b_3 = 1.184616 \\ 32 \div 27 &= c_3 = 1.185185 \\ c_3 - b_3 &= d_3 = .000569 \\ c_3 - a_3 &= e_3 = .000166 \end{aligned}$$

$e_3 \div d_3 = \frac{166}{569}$, the probable error being only $\pm \frac{142}{569}$. Therefore the test fails in this case.

The ratio between β and γ is between $\frac{28}{25}$ and $\frac{75}{67}$.

$$\begin{aligned} \beta \div \gamma &= a_4 \doteq 1.119816 \\ 28 \div 25 &= b_4 = 1.120000 \\ 75 \div 67 &= c_4 = 1.119403 \\ b_4 - c_4 &= d_4 = .000597 \\ b_4 - a_4 &= e_4 = .000184 \end{aligned}$$

$e_4 \div d_4 = \frac{184}{597}$, the probable error being only $\pm \frac{149}{597}$. Therefore the test fails in this case also.

The ratio between α and γ is between $\frac{62}{41}$ and $\frac{65}{43}$.

$$\begin{aligned} \alpha \div \gamma &= a_5 = 1.511936 \\ 62 \div 41 &= b_5 = 1.512195 \\ 65 \div 43 &= c_5 = 1.511628 \\ b_5 - c_5 &= d_5 = .000567 \\ b_5 - a_5 &= e_5 = .000259 \end{aligned}$$

$e_5 \div d_5 = \frac{37}{81}$, the probable error being only $\pm \frac{20}{81}$. The test, therefore, fails again, the number of failures in the whole comparison being equal to the number of confirmations. Hence it is evident that Schuster's criterion is insufficient, at least when the probable errors of observation are not satisfactorily ascertained. Even if the probable errors were known, the proper application of the test would require supplementary calculations of such intricacy as to make it practically inoperative.

146. *Modifications of the Test.*

By increasing the magnitude of the harmonic ratios the test may sometimes be made to indicate a probability. For example, $\beta \div \gamma$ is between $\frac{6}{5}$ and $\frac{7}{6}$. These values give $d_4 = .033333$; $e_4 = .000184$; $e_4 \div d_4 = \frac{184}{33333}$, which is less than $\frac{1}{45}$ of the probable error. In like manner $\alpha \div \gamma$ is between $\frac{3}{2}$ and $\frac{4}{3}$. These values give $d_5 = .166667$; $e_5 = .011936$; $e_5 \div d_5 = \frac{1119}{15625}$, which is less than $\frac{4}{13}$ of the probable error. These results seem to indicate the propriety of harmonic comparisons between terms which are unquestionably of the same order of magnitude. Thus in Schuster's calculation (loc. cit., p. 338), the ratio .96476 lies between $\frac{55}{57}$ and $\frac{55}{58}$, the difference between these two fractions being .016636. The difference of the fraction in the sodium spectrum from the nearest of these comparative fractions is .000152, which is only .00914 of the difference between the fractions themselves, or less than $\frac{1}{27}$ of the probable error.

If a supposed harmonic relation can be represented by a fraction with terms of a single digit, Schuster's test might fail even with the above modification, provided the probable error should be $> \frac{1}{4} \times \frac{1}{8} \times \frac{1}{9}$; if the terms are of two digits, it would not be trustworthy if the probable error was $> \frac{1}{4} \times \frac{1}{98} \times \frac{1}{99}$. If the modifications of *vis viva* in synchronous wave systems are of the same order of magnitude as the variations of planetary eccentricity, the limit of probable error would be at least $\frac{1}{3}$, instead of $\frac{1}{4}$, of the difference between adjacent fractions which have a common numerator. This would be the case for each of the compared pairs of wave lengths, the probability for the entire system being equivalent to the product of all the independent probabilities.

All estimates of abstract probability, in such cases, should be greatly increased by the *a priori* probability, or even the mathematical necessity, that synchronous undulations in elastic media *must be* harmonic. In view of this consideration, the indications of a harmonic tendency pervading an entire system, such as I have pointed out in many of my comparisons, are far more significant than any conclusions that can be drawn from more restricted investigations.

Whatever test may be applied, it should always be remembered that the failure to discover a harmonic influence between any two given lines does not affect, in the slightest degree, the evidences of harmonic influence between other lines. The failing cases are entitled to no weight in drawing the final conclusion. We should, therefore, have been justified in stopping our examination of the observed lines in the hydrogen spectrum, as soon as we found that α is harmonically connected with β and δ, and that γ is similarly connected with δ. Even if subsequent discussions had failed to show any probable evidence of harmony between β and γ, β and δ, α and γ, the fact that there are such harmonies operating through the relations of the intermediate to the extreme wave-lengths, would have been unshaken.

147. *Uncertainties of Measurement.*

The influence of probable errors of observation upon the validity of Schuster's criterion may be illustrated by applying it to two of the different values which have been found for the Fraunhofer lines A, B and C. Angstrom's measurements are taken from *Schellen's Spectrum Analysis*, p. 168; Gibbs's, from *Am. Jour. Science*, [2] xliii, 4.

	Gibbs.		Angstrom.
A	761.20		760.09
B	687.49		686.68
C	656.77		656.18
$A \div B = a$	1.107216		1.106906
$b = 31 \div 28$	1.107143	$31 \div 28$	1.107143
$c = 72 \div 65$	1.107692	$83 \div 75$	1.106666
$c - b = d$	.000549	$b - c$	.000477
$a - b = e$	.000073	$b - a$	.000237
$e \div d$	$73 \div 549$		$79 \div 159$
Probable error	$137 \div 549$		$39 \div 159$
$B \div C = a_1$	1.046776		1.046481
$b_1 = 45 \div 43$	1.046512	$45 \div 43$	1.046512
$c_1 = 67 \div 64$	1.046875	$68 \div 65$	1.046154
$c_1 - b_1 = d_1$	.000363		.000358
$c_1 - a_1 = e_1$	.000109	$b_1 - a_1$	.000031
$e_1 \div d_1$	$109 \div 363$		$31 \div 358$
Probable error	$90 \div 363$		$89 \div 358$

Hence the criterion indicates a harmony of vibration, both between the A and B lines, according to Gibbs, and between the B and C lines, according to Schellen. The *a priori* probability or certainty that there must be such a harmony, lends confidence to the greater accuracy of Gibbs's measurement of the A line, and of Schellen's measurement of the C line. If such allowance as I have proposed is made for that probability, the harmony is shown by Gibbs in both comparisons.

148. *The Fraunhofer Harmonies.*

The general accuracy of Gibbs is confirmed by the fact that his measurements indicate harmonies among the principal Fraunhofer lines, as is shown in the following table. They all bear the test of Schuster's criterion, with the exception which was stated in the foregoing note.

Harmonies.	Gibbs.	Difference.
A 761.20	A 761.20	
$\frac{28}{31}$ A 687.54	B 687.49	.05
$\frac{64}{67}$ B 656.75	C 656.77	.02
$\frac{44}{49}$ C 589.73	D_γ 589.74	.01
$\frac{76}{85}$ D 527.29	E 527.38	.09
$\frac{89}{72}$ H_2 486.68	F 486.52	.16
$\frac{23}{21}$ H_2 431.24	G 431.03	.21
$\frac{15}{29}$ A 393.72	H_2 393.59	.13

The greatest difference is only $\frac{1}{20}$ of one per cent., in the G line.

149. *Proper Use of Harmonic Tests.*

In systems of waves which are propagated with such frequency as the undulations of light, it may, perhaps, be impossible to devise any criterion which will show whether any two given waves are really harmonic. But if we consider that undulations which are not harmonic are continually tending to destroy each other, various useful tests may be found, which will serve as guides for the approximate determination of harmonies that must really exist. For example, if there are two harmonic light-waves, the slower oscillating 1670 times, while the swifter oscillates 1843 times, there will be more than 300,000,000,000 coincidences of phase per second, and yet Schuster's method would lead us to suppose that there is no harmony. The ratio $\frac{1670}{1843}$, is equivalent to .906131, which is between $\frac{29}{32}$ and $\frac{77}{85}$.

29 ÷ 32	.906250
77 ÷ 85	.905882
Difference	.000368
.906250 — .906131	.000119
Ratio of Differences 119 ÷ 368	.324
Probable Error	.250

150. *Successive Harmonies.*

The following table shows that Angstrom's measurements indicate harmonic undulations, which present more than 600,000,000,000 coincidences of phase per second in successive lines :

Wave-length.	Log. w. l.	Log. ratio,	Ratio.
A 760.09	2.8808650	$\bar{1}$.9755619	501 / 530
a 718.50	2.8564269	$\bar{1}$.9803266	410 / 429
B 686.68	2.8367535	$\bar{1}$.9802695	710 / 743
C 656.18	2.8170230	$\bar{1}$.9534617	389 / 433
D_1 589.50	2.7704847	$\bar{1}$.9512343	446 / 499

Wave-length.	Log. w. l.	Log. ratio,	Ratio.
E. 526.89	2.7217190	$\bar{1}.9919387$	427 / 435
b_2 517.20	2.7136577	$\bar{1}.9730311$	359 / 382
F 486.06	2.6866888	$\bar{1}.9475091$	288 / 325
G 430.72	2.6341979	$\bar{1}.9787118$	418 / 439
h 410.12	2.6129097	$\bar{1}.9856655$	149 / 154
H_1 396.80	2.5985752	$\bar{1}.9961108$	667 / 673
H_2 393.26	2.5946860		

Schellen's table gives 393.28 for the wave-length of H_2. His other values are precisely the same as the harmonic lengths which are given here. The greatest interval between the successive coincidences of phase in the propagation of the waves which represent any two adjacent lines, is less than $\frac{1}{50}$ of an inch.

Some of the closer lines precisely represent simple geometric progressions within the limits of uncertainty of observation. The middle D line, according to Gibbs, is a geometrical mean between the extreme lines.

Gibbs.	Geometric.	Log.
β 590.04	590.038	2.7708800
γ 589.74	589.736	2.7706576
α 589.43	589.434	2.7704352

By making $r = \frac{953}{952}$, we find that the three b lines are in geometrical progression :

Schellen.	Geometric.	Log.
b_1 518.30	ar_3 518.296	2.7145778
b_2 517.20	ar 517.209	2.7136658
b_3 516.67	a 516.666	2.7132098

Gibbs, in *Johnsons' Cyclopedia,* gives four b lines, the additional line being also geometrically determined :

Gibbs.	Geometric.	Log.
b_1 518.31	ar^3 518.31	2.7145896
b_2 517.22	ar 517.22	2.7136776
b_3 516.85	$ar^{\frac{1}{3}}$ 516.86	2.7133736
b_4 516.69	a 516.68	2.7132216

151. *Conjoined Harmonies.*

The indications of geometrical progression in the D and b groups suggest the propriety of looking for similar evidence among the remaining lines. The relations which I have pointed out between æthereal and planetary nodes (Note 37), led me to look to the inertia of Sun and Jupiter as an important source of nodal influence upon æthereal waves. Light-waves would traverse a trajectory equivalent to that of Earth's daily synodic rotation 5.202798 × 365.25636 = 1900.355 times, during the interval which would be required for them to traverse one equivalent to Jupiter's orbit. If we suppose the length of each wave to be increased in the ratio $r = (1901.355 \div 1900.355)^{\frac{1}{4}}$, we get the following approximations :

Geometric wave-lengths.		Log. w. l.	Log. r^n.	Schellen.
$A = ar^{428}$	760.10	2.8808707	.0244495	760.09
$a = Br^{344}$	718.49	2.8564212	.0196510	718.50
$B = Cr^{345}$	686.70	2.8367702	.0197081	686.68
$C = D_1 r^{815}$	656.24	2.8170621	.0465569	656.18
$D = Er^{854}$	589.53	2.7705052	.0487848	589.50
$E = b_2 r^{141}$	526.89	2.7217204	.0080546	526.89
$b_2 = Fr^{472}$	517.21	2.7136658	.0269630	517.20
$F = Gr^{919}$	486.07	2.6867028	.0524979	486.06
$G = hr^{373}$	430.73	2.6342049	.0213076	430.72
$h = H_1 r^{251}$	410.11	2.6128973	.0143384	410.12
$H_1 = H_2 r^{68}$	396.79	2.5985589	.0038845	396.80
H_2	393.26	2.5946744		393.28

The lines can be grouped in three sets :—1. A, a, B, D_1 ; 2. C, b_2, F, h ; 3. E, G, H_1, H_2. All members of the first and third groups are connected with the other members of their own groups by some power of r^4 ; those of the second group, by some power of r^2 ; the members of the third group being connected with those of the first by some odd power of r^2 ; those of the second with those of either of the other groups, by odd powers of r. Three lines of the middle group, C, F, h, are hydrogen lines. The boundaries of the group are phyllotactic ; 5 C = 8 h.

152. *Interpretation and Mass-Relations.*

The considerations involved in the foregoing notes were orbital velocity $\left(v_0 \propto \sqrt{\frac{1}{r}}\right)$, gravitating velocity, $\left(v_1 = g \propto \frac{1}{r^2}\right)$, the constant velocity of light, (v_2) and inertia, or mass. Orbital velocity varies as the fourth root of gravitating velocity, $\left(v_0 \propto v_1^{\frac{1}{4}}\right)$; hence the ratio of increase varies as the fourth root of the relative disturbance of inertia, $r = (1901.355 \div 1900.355)^{\frac{1}{4}}$.

If we take the constant relation between gravitating and photodynamic action at Earth's surface as a unit, ($p_3 = 1$), there should be some harmonic relation between p_3 and Jupiter's incipient perturbation, ($p_5 = r^4 — 1$), dependent on the masses of Earth, (m_3), and Jupiter, (m_5). Earth's reaction upon its linear centre of oscillation, relative to the centre of the "line of force" which connects it with Jupiter, is exerted at ($\frac{1}{3}$ of $\frac{1}{2}$) = $\frac{1}{6}$ of the distance of Jupiter's action upon Earth, consequently with a six-fold relative efficiency. Accordingly we find :

$$\frac{m_3}{m_5} = \frac{6\,p_5}{p_3} = \frac{6}{1900.355} = \frac{1}{316.726}$$

$$\frac{m_0}{m_3} = 316.726 \times 1047.879 = 331890$$

$$\rho_3 = 92,796,300 \text{ miles.}$$

This value of ρ_3 differs by less than $\frac{1}{15}$ of one per cent. from the mean of various mechanical estimates, which was given in Note 15.

153. *Linear Harmonies.*

The harmonies to which Schuster refers, in the third conclusion of his summary, seem to be linear rather than geometric. If we omit the specially phyllotactic lines C and *h*, together with the secondary lines of the D and *b* groups, the percentages of the geometric, Schellen and Gibbs tables give the following coincidences of harmonic tendency :

	Geometric.		Schellen.		Gibbs.	
A	760.10	1933 — .16	760.09	1933 — .31	760.40	1933 + .38
a	718.49	1827 + .03	718.50	1827 — .06	718.47	1827 — .23
B	686.70	1746 + .21	686.68	1746 + .03	686.71	1746 + .02
D	589.53	1499 + .10	589.50	1499 — .07	589.51	1499 — .12
E	526.89	1340 — .18	526.89	1339 — .27	526.96	1340 — .16
b_2	517.21	1315 + .19	517.20	1315 + .09	517.22	1315 + .08
F	486.07	1236 + .03	486.06	1236 — .09	486.07	1236 — .12
G	430.73	1095 + .29	430.72	1095 + .20	430.73	1095 + .17
H_1	396.79	1009 — .02	396.86	1009 — .05	396.81	1099 — .02
H_2	393.26	1000	393.28	1000	393.30	1000

The only instances in which the deviations from exact harmony exceed $\frac{1}{4}$ of the right hand unit, are the G line in the geometric table, the A and E lines of Schellen, and the A line of Gibbs.

154. *Interstellar Phyllotaxy.*

The interstellar abscissas, Note 46, present the following phyllotactic features :

1. The determination of the paraboloid requires 3 abscissas.
2. The whole number of abscissas, between the vertex and the stellar region, is 3×13.
3. The abscissas between A_2 and A_{39} are divided into two groups, each group containing 2×3^2 abscissas.
4. Each group has two equal subdivisions ; the inner representing tendencies to condensation, the outer giving no present evidence of such tendency, except in comets, meteors and possibly asteroids.

155. *Distance of α Centauri and its connection with the Interstellar Paraboloid.*

Newcomb gives estimates of the parallax of *α Centauri*, ranging between 0″.48 and 1″.96. The mean of Henderson's observations, in 1832–3, as deduced by himself, was 1″.16 ± .11. Peters, from the same observations, found 1″.14 ± .11. Henderson obtained 0″.913 from Maclear's observations in 1839–40 ; Peters, 0″.976 ± .064 from the same ; Maclear, 0″.919 ± .034 from declinations in 1842, 1844 and 1848 ; Moesta, 0″.880 ± .068 from declinations in 1860–4. There is, therefore, an uncertainty as to the actual distance, which is of the same order of magnitude as planetary eccentricities. If this fact should be thought to diminish the probability of a kinetic bond between the photodynamic paraboloid and the fixed stars, it will be well to bear in mind the following considerations :

1. If there is an all pervading interstellar medium, which is both material and elastic, all its persistent oscillations *must be* cyclically harmonic in some way or other.

2. All such permanent oscillations must be dependent upon or associated with permanent masses and velocities.

3. The mass and velocity from which the paraboloidal abscissas were deduced, are the mass of the sun and the velocity of light.

4. The coördinates indicate a solar motion in space, which is closely accordant with Herschel's estimated velocity. (Note 112.)

5. The abscissas locate regions of incipient subsidence, which account for the formation of the several planetary belts, in accordance with Herschel's interpretation of the nebular hypothesis.

6. The abscissas are manifoldly grouped, in ways that are phyllotactically and otherwise harmonically symmetrical, as might be looked for in a medium like the supposed luminiferous æther.

7. The last phyllotactic abscissa, A_{38}, is a fourth proportional to Sun's radius, Laplace's limit, and the solar modulus of light.

8. The paraboloid fixes Sun's position, relatively to some other important stars in the Milky Way. (Note 114.)

9. These are the most far-reaching indications of an unbroken chain of kinetic influences, that have ever been published.

10. Being based upon the greatest mass and the greatest intercosmical velocity of which we have any measurable knowledge, the law of parsimony gives an *a priori* presumption that the chain may extend to other masses of the same order of magnitude as the Sun.

11. The next abscissa to the solar phyllotactic series, A_{39}, is in the region of the fixed stars, its locus being, within the limit of probable error, ($\pm$ .25), the same as that of *α Centauri.*

12. The terminal locus is not only within, but *far* within, the limits of probable error. Its accordance with *α Centauri* may be exact ; it is almost impossible that the deviation from precise accordance can be so great as 8 per cent., and such a deviation could be easily explained by stellar orbital motions.

13. The second stellar abscissa, A_{40}, indicates a distance corresponding to Bessel's estimate of the parallax of 61 *Cygni.*

14. Whatever may be thought of the last three indications, the first ten are plain, unmistakable and incontrovertible.

156. *Correlations of Planetary Mass and Distance.*

Stockwell closes the introduction to his "Memoir on the Secular Variations of the Elements of the Orbits of the Eight Principal Planets," in the following words :

"The idea is thus suggested of the existence of a system of bodies in which the masses of the different bodies are so adjusted to their mean distances as to insure to the system a greater degree of permanence than would be possible by any other distribution of masses. The mathematical expression of a criterion for such distribution of masses has not yet been fully developed; and the pending illustrations have been introduced here, more for the purpose of calling the attention of mathematicians and astronomers to this interesting problem than for any certain light we have yet been able to obtain in regard to its solution."

When I began my investigation of the harmonies which illustrate the laws of æthereal elasticity, the only published evidence of any connection between planetary mass and distance was Alexander's approximate equality between the products of the masses of Jupiter and Saturn by the squares of their respective major axes:

$$\text{Jupiter } \tfrac{1}{1047.879} \times 5.202798^2 = .025832.$$
$$\text{Saturn } \tfrac{1}{3501.6} \times 9.538852^2 = .025985.$$

Laplace, however, had indicated elements of stability in the sums of various products, and had demonstrated the tendency of approximate synchronisms to become exact; Herschel had shown that "subsidence, and the central aggregation consequent on subsidence, may go on quite as well among a multitude of discrete bodies under the influence of mutual attraction and feeble or partially opposing projectile motions, as among the particles of a gaseous fluid"; * various physical investigations, based upon propositions in Newton's Principia, had indicated the fact that *all persistent oscillations or cyclical motions in elastic media* MUST BE *subject to harmonic laws.*

In studying Herschel's statements of the nebular hypothesis, it soon became evident that Sun, at the principal centre of nucleation, Earth, at the centre of the belt of greatest condensation, and Jupiter, at the centre of the primitive nebula, had some important common relations which had exerted a controlling influence over the other cosmical masses. The four following are specially noteworthy.

1. The "nascent velocity" of Sun, $\frac{gt}{2}$†, *is equivalent to the velocity of light.*

2. The nascent velocities of Earth and Jupiter are nearly equal.

3. Earth's nascent velocity is about 3 per cent. less, while Jupiter's appears to be slightly greater, than the limit of possible circular-orbital velocity, $\sqrt{gr}$ at Sun's surface.

4. The aggregate orbital *vis viva* of Earth, vv_3, and Jupiter, vv_5, is a simple function of mean distance from Sun, d, and orbital time, $d^{\frac{3}{2}}$.

$$\left(\frac{d_5}{d_3}\right)^{\frac{5}{2}} = \left(\frac{5.202798}{1}\right)^{\frac{5}{2}} = 61.7436 = 1 + 60.7436 = vv_3 + vv_5 \quad (1)$$

*Outlines of Astronomy, Sect. 871.

† t being time of solar rotation, and g being the acceleration of superficial gravitation.

In the above equation, and elsewhere in the present note, Earth's mass and semi-axis major are taken as the units of mass and distance.

In studying the results of nebular "subsidence," let α represent the locus of incipient belt-subsidence or secular aphelion ; β, the locus of mean belt subsidence or mean aphelion ; γ, the semi-axis major ; δ, the mean locus of belt-rupture or mean perihelion ; ε, the locus of incipient belt-rupture or secular perihelion ; subscript 0, 1, 2, 3, 4, 5, 6, 7, 8, Sun and the principal planets in their order, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune ; m, mass.

$$m_0 : m_5 : : \varepsilon_5 : \varepsilon_0 : : 1047.879 : 1 \qquad (2)$$

According to Stockwell (op. cit. p. 38), $\varepsilon_5 = .9391726\,\gamma_5$. Therefore $\gamma_3 = 1047.879\ \varepsilon_0 \div (5.202798 \times .9391726) = 214.4513\ \varepsilon_0$ and $\gamma_5 = 1115.74\ \varepsilon_0$. Mass being represented by the product of mean orbital *vis viva* by mean distance, we find, from (1) and (2),

$$m_3 : m_5 : : 1 \times 1 : 60.7436 \times 5.202798 : : 1 : 316.0366 \qquad (3)$$

$$m_0 : m_3 : : (316.0366 \times 1047.879 = 331668) : 1 \qquad (4)$$

Earth's chief companion planet, Venus, shows the influence of incipient subsidence-aggregation.

$$m_3 : m_2 : : \gamma_3 : \alpha_2 : : 1 : .77442 \qquad (5)$$

$$m_0 : m_2 : : (331668 \div .77442 = 427630) : 1 \qquad (6)$$

Jupiter's chief companion planet, Saturn, shows the combined influence of modified belt-rupture and secular stability.

$$m_5 : m_6 : : \beta_6\gamma_6 : \beta_5\gamma_5 : : 9.077645 \times 9.538852 : 4.978245 \times 5.202798 : : 3.3432 : 1 \qquad (7)$$

$$m_0 : m_6 : : (3.3432 \times 1047.879 = 3503.22) : 1 \qquad (8)$$

The outer planet of the dense belt, Mars, shows the influence of modified subsidence at the centre of nebular planetary inertia (Saturn), combined with that of incipient subsidence at the centre of the belt (Earth).

$$m_3 : m_4 : : \beta_6 : \alpha_3 : : 10 : 1.0677352 : : 9.3657 : 1 \qquad (9)$$

$$m_0 : m_4 : : (9.3657 \times 331668 = 3101613) : 1 \qquad (10)$$

The two outer planets of the system show the combined influence of nodes of subsidence, rupture, and condensation.

$$m_8 : m_3 : : (\delta_8 - \alpha_3)\gamma_8 : (\delta_8 + \alpha_7)\gamma_3 : : (29.73235 - 1.06774) \times 30.03386 : (29.73235 + 20.679233) : : 17.0776 : 1 \qquad (11)$$

$$m_0 : m_8 : : (331668 \div 17.0776 = 19392) : 1 \qquad (12)$$

$$m_7 : m_3 : : (\beta_7 - \varepsilon_3)\,\gamma_7 : (\beta_7 + \delta_5)\,\gamma_3 : : (20.04418 - .93226) \times 19.18358 : (20.04418 + 4.97824) : : 14.6523 : 1 \qquad (13)$$

$$m_0 : m_7 : : (331668 \div 14.6523 = 22602) : 1 \qquad (14)$$

The accordance between the nodal and the computed values is shown in the following table :

		Nodal.	Computed.	Authority.
(2)	$m_0 \div m_5$	1047.879	1047.879	Bessel.
(4)	$m_0 \div m_3$	331668.	331776.	Chase.
(6)	$m_0 \div m_2$	427630.	427240.	Hill.
(8)	$m_0 \div m_6$	3503.22	3501.6	Bessel.
(10)	$m_0 \div m_4$	3101613.	3093500.	Hall.
(12)	$m_0 \div m_8$	19392.	19380.	Newcomb.
(14)	$m_0 \div m_7$	22602.	22600.	Newcomb.

The mean orbital *vis viva* which could be communicated to a subsiding particle in any given time, varies directly as the mean gravitating acceleration, $\left(\frac{1}{\gamma}\right)^2$, and inversely as the mean subsidence potential, $\left(\frac{1}{\beta}\right)$. The synchronism in the determination of the *vis viva* of the two chief planets is shown by the proportion

$$m_5 \gamma_6 : m_6 \gamma_5 : : \gamma_6^2 \beta_6 : \gamma_5^2 \beta_5 \qquad (15)$$

which is readily deduced from (7). The confirmation which this lends to Herschel's interpretation of the nebular hypothesis is very satisfactory,

The planets of the dense belt bear witness to an interesting combination of harmonic influences. The simple attraction of a principal centre of condensation would tend to produce radial oscillations and very elliptical orbital paths. If the central force could be concentrated in a point, the elliptic path would coincide with the major-axis and be rectilinear, its orbital time being $2\pi\sqrt{\frac{r}{g}}$ and the mean velocity being $\frac{2}{\pi}\sqrt{gr}$; g representing the acceleration of the central force at the distance r. If the radial oscillations are made circular, (through the mutual collision of particles, the exchange of molecular *vis viva* for *vis viva* of rotation or of revolution, external attraction and other physical disturbances), the orbital velocity should be some function of the nascent velocity, $\frac{gt}{2}$.

The combined orbital *vis viva* of Earth and Jupiter, (1), suggests the consideration of nebular subsidence from Moon's apogee towards the mean centre of gravity of Jupiter and Sun. In expanding or contracting nebulæ, rotating velocity and circular orbital *vis viva* both vary inversely as radius. In considering ultimate particles or chemical atoms of equal volume, the mass and the limiting or maximum acceleration both vary as the density ; the limit of gravitating *vis viva*, therefore, varies as the cube of the density, ($\delta \times \delta^2 = \delta^3$) and centrifugal *vis viva* varies as $\frac{1}{\delta^3}$. Circular orbital or tangential atomic *vis viva* varies directly as density and inversely as radius-vector, $(\delta \div \rho)$. Equating these two expressions for centrifugal energy, at the centre of condensation, we get

$$\frac{1}{\delta^3} \propto \frac{\delta}{\rho} \qquad (16)$$

$$\delta \propto \rho^{\frac{1}{4}} \propto \frac{t^2}{\rho^3} \qquad (17)$$

$$t^2 \propto \rho^{\frac{13}{4}} \propto \delta^{13} \qquad (18)$$

The limit of circular-orbital time, $2\pi\sqrt{\frac{r}{g}}$, for Earth, is $t_3 = 5073.6$ seconds, Earth's orbital time being $t_1 = 1$ year $= 31558149$ seconds. If these times were fixed by the above relations of "subsidence" we should have

$$31558149^2 : 5073.6^2 : : \delta_a^{13} : \delta_o^{13} \qquad (19)$$

$$\delta_a = 3.8341\ \delta_o \qquad (20)$$

$$\rho_a : \rho_o : : \delta_a^4 : \delta_o^4 : : 216.0944 : 1 \qquad (21)$$

Earth's present density, as deduced from the combined influences of subsidence, linear oscillation, conical oscillation and orbital velocity, (Notes 5, 23), is $3.9231\ \delta_o$, as is shown by the following calculations :

$$m_o = (2 \times 3 \times 4)^4\ m_3 = 331776\ m_3 \qquad (22)$$

$$\rho_3 = (m_o \div m_3)^{\frac{1}{3}} \times (t_1 \div t_3)^{\frac{2}{3}} \times r_3 = 92,785,700 \text{ miles.} \qquad (23)$$

$$\rho_o = \rho_3 \div 214.4472^* = 432673.8 \text{ miles.} \qquad (24)$$

$$\left.\begin{array}{l} \delta_3 : \delta_0 : : m_3 \div r_3^3 : m_o \div \rho_o^3 : : 3.9231 : 1 \\ \text{or, from (20), (21) and (22), } \delta_3 : \delta_a : : 216.0944^3 : 214.4472^3 \\ \qquad : : 3.9231 : 3.8341 \\ \delta_3 : \delta_o : 3.9231 : 1 \end{array}\right\} \qquad (25)$$

In (21) the radius of early lunar-telluric subsidence, ρ_a, includes:— α. Moon's orbital radius of incipient subsidence, or locus of apogee, $.5824\ \rho_o$; β. Earth's semi-axis major, $214.4472\ \rho_o$, (24); and γ. The semi-axis major of the centre of gravity of Sun and Jupiter, $1.0647\ \rho_o$.

α. According to Von Littrow, Moon's locus of apogee $= 1.054908 \times 60.2776 \times 3962.8 = 251,985$ miles $= .5824\ \rho_o$, (24).

γ. According to Stockwell, (Smithsonian Contrib., 232, p. 38), Jupiter's maximum eccentricity is .0608274. This gives, for the semi-axis major of the chief centre of gravity of the solar system, (Note 113), $\rho_o \div .9391826 = 1.064767\ \rho_o$.

The influence of nascent *vis viva*, at the centre of the belt of greatest condensation, is further shown by the relations which are maintained, with close approximation, between Earth and its adjacent planets.

Nascent velocity at Laplace's limit, and consequent orbital *vis viva*, is

* See Note 113.

π times as great as velocity of rotation at the same limit. This leaves π — 1 units of Earth's subsident rotating velocity to be converted into orbital *vis viva*. Taking Earth's mass and semi-axis major as the units of mass and distance, the foregoing relative theoretical values of nodal mass yield the following comparative results :—

	Orbital *Vis Viva* = Mass ÷ Rad. Vect.	Theoretical *Vis Viva*.	
Earth	$1 \div 1 = 1.$	$1.$	(26)
Venus	$(331668 \div 427630) \div .72333 = 1.0723$	$(\pi - 1) \div 2 = 1.0708$	
Mars	$(331668 \div 3101613) \div 1.52369 = .0702$	$(\pi - 3) \div 2 = .0708$	
	2.1425	$\pi - 1 \quad 2.1416$	

The influences of *vis viva* which are represented by the two outer planets, (11) (13), are somewhat more intricate, but no less interesting than those of their companions. Orbital velocity, in the path of any given planet, varies inversely as radius vector, following the same law as variations of rotating velocity in an expanding or contracting nebula, and as variations of orbital *vis viva* in revolving particles at different distances from a controlling centre. Hence there are tendencies to maximum and mean accelerations and retardations at secular and mean apsides, which are shown in the amounts of nodal planetary aggregation that are required to maintain the cyclical equilibrium of orbital *vis viva*. The ratio of Neptune's *vis viva* to the mass at the chief centre of condensation, Earth, is determined by cardinal nodes of Neptune, Uranus and Earth ; the like ratio of the *vis viva* of Uranus, by cardinal nodes of Uranus, Jupiter and Earth. Earth's nodes are those of incipient subsidence and rupture at the centre of density, secular aphelion and perihelion ; the node of Uranus which is influential upon Neptune is that of incipient subsidence ; the three other nodes are of mean subsidence or rupture. Earth's secular aphelion has modified the aphelion planet, while its secular perihelion has modified the perihelion planet of the outer two-planet belt.

Atomic phyllotaxy, times and acquired velocities of subsidence, orbital velocities resulting from subsident *vis viva*, and the relations of density to distance from a controlling centre in an elastic medium, are all illustrated by the *exact* equality, (21) (23) :—

$$(t_1 \div t_3)^{\frac{8}{13}} = \rho_a \div \rho_o \qquad (27)$$

The order of importance of the equilibrating manifestations of cosmical *vis viva* in the solar system, seems to be the following :—

1. The relation of Sun's mass to mass at principal centre of density, Earth, which satisfies necessary tendencies to subsidence, oscillation, and orbital revolution (22).

2. The combined orbital *vis viva* at the chief centre of density and at the centre of primitive nebulosity (1).

3. The incipient tendency to rupture, between the chief centres of nucleation and of nebulosity, Sun and Jupiter (2).

4. The relations of *vis viva* between the chief centres of density and of nebulosity, Earth and Jupiter (3).

5. The relations between mean tendencies to belt-rupture and to secular stability, between the chief centres of nebulosity and of planetary inertia, Jupiter and Saturn, in the chief belt of planetary aggregation (7).

6. The tendency of incipient subsidence towards the chief centre of nucleation, and of incipient rupture from the chief centre of density, Venus, Sun and Earth (5).

7. The tendency of incipient subsidence at the chief centre of density, together with mean tendencies of subsidence at the chief centre of planetary inertia, Earth, Saturn and Mars (9).

8. The combination of incipient tendencies of subsidence at the chief centre of density and at the inner margin of the outer planetary belt, with mean rupturing tendencies at the outer margin of the outer belt (11).

9. The combination of incipient tendencies of rupture at the chief centre of density, with mean rupturing tendencies at the chief centre of nebulosity and mean subsidence at the inner margin of the outer belt (13).

Mercury, with its great orbital eccentricity, the asteroids, comets and meteors, doubtless serve to complete the exact adjustment which the stability of the system requires.

157. *Relations of Fraunhofer Lines to Density and Vis Viva.*

Astronomers do not waste their time in inquiring whether planetary motions are in accordance with the laws of gravitation, neither need physicists ask whether cyclical undulations are harmonic. Knowing that they must be so, the wiser way is to question nature in order to find what the harmonies are. The simplest harmonies are those which are based on multiples of 2 or 3. If we take $\lambda = 392.78$ as a unit, we find the following approximation of wave-lengths, as measured by Gibbs, to harmonic values :

Harmonic.			Observed.	Error.	Probable Error.
$\frac{7}{4}\lambda$		$= 687.37$	B 686.71	$+0.66$	± 24.54
	$\frac{5}{3}\lambda$	$= 654.63$	C 656.21	-1.58	± 32.73
$\frac{6}{4}\lambda$		$= 589.17$	D_1 589.51	-0.34	± 24.54
	$\frac{4}{3}\lambda$	$= 523.71$	E 526.87	-3.16	± 32.73
$\frac{5}{4}\lambda$		$= 490.97$	F 486.07	$+4.90$	± 24.54
$\frac{4}{4}\lambda$	$\frac{3}{3}\lambda$	$= 392.78$	H_2 393.30	-0.52	± 32.73

The probable error, in each instance, is one-fourth of the harmonic divisor, in accordance with Schuster's proposed test. The greatest discrepancy, F, is only $\frac{1}{5}$ of the probable error, or only $\frac{1}{5}$ as great as we might look for without invalidating the evidence of harmonic influence. Such accordance is surely satisfactory enough to encourage further examination.

If we take $l = \frac{1}{24}\lambda$ as a unit, so as to provide for the requirements of centripetal, linear and conical oscillation, ($2 \times 3 \times 4$; see Notes 5, 23, 156), we find the following approximations :—

Harmonic.	Observed.	Error.
$46\,l = 752.83$	A 760.40	—7.57
$44\,l = 720.10$	a 718.47	+1.63
$42\,l = 687.37$	B 686.71	+0.66
$40\,l = 654.63$	C 656.21	—1.58
$36\,l = 589.17$	D_1 589.51	—0.34
$32\,l = 523.71$	E 526.87	—3.16
$30\,l = 490.97$	F 486.07	+4.90
$26\,l = 425.51$	G 430.73	—5.22
$25\,l = 409.15$	h 410.12	—0.97
$24\,l = 392.78$	H_2 393.30	—0.52

If the Fraunhofer lines are, like musical beats, due to the interference of waves which are very nearly but not exactly in unison, their proper investigation requires a consideration of more intricate harmonies than those which are based upon simple binary or ternary division. The harmonic A line seems to be attracted towards the simple octave of H_2, and H_1 indicates a reaction resulting from such attraction. For, if H_1 be divided into 12 parts, (3×4), and H_2 into 15 parts, (3×5), each of the submultiples will also be a submultiple of A, very nearly, if not exactly.

Harmonic.	Observed.	Error.	
$A_0 = 760.38$	A 760.40	+.02	
$\chi_1 = 396.72$	H_1 396.81	—.09	(1)
$\chi_2 = 393.30$	H_2 393.30	.00	

$\chi_1 = \frac{12}{23} A_0$; $\chi_2 = \frac{15}{29} A_0$.

The line b_1 is very nearly $\frac{4}{3}$, or more nearly $\frac{64}{49}$ of H_1; the lines b_4, E, D_1, are very nearly $\frac{21}{16}$ (or about $\frac{5}{4}$), $\frac{75}{56}$ (or about $\frac{4}{3}$), and $\frac{3}{2}$ of H_2, respectively. In other words, H_1, b_1, A are nearly enough in the simple harmonic ratio 3, 4, 6, to produce luminous beats, or dark lines, while H_2, b_4, E, D_1, A, show a like approximation to 12, 15, 16, 18, 24.

The tendency of the harmonic ratios to become simply geometric, (Notes 41–43), is illustrated by approximations to the following equations: $B = \sqrt{a\,C}$; $D_1 = \sqrt{a\,F}$; $D_2 = \sqrt{C\,E}$; $b_1 = \sqrt{C\,h}$; $b_2 = \sqrt{b_1\,b_4}$; $H_1 = \sqrt{h\,H_2}$. The first of these equations gives the following accordances:

Harmonic.	Observed.	Error.	
$\alpha = 718.51$	a 718.47	+.04	
$\beta = \sqrt{\alpha\gamma} = 686.67$	B 686.71	—.04	(2)
$\gamma = 656.25$	C 656.21	+.04	

From (1) and (2) the harmonic series in the following table is constructed by using the exact harmonic ratios $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{5}{8}$, $\frac{6}{7}$, together with the disturbing ratios $\frac{49}{64}$, $\frac{56}{75}$, $\frac{16}{21}$, (which are all near enough to $\frac{3}{4}$ to produce luminous beats), and $\frac{27}{32}$ which is nearly equivalent to $\frac{5}{6}$. Four of the numbers which appear as terms of the exact harmonic ratios, (2, 3, 5, 8), belong to the phyllotactic series; one, $(6 = 2 \times 3)$, is the product of two adjacent phyllotactic factors; one, 7, is a prime number.

Harmonic.	Observed.	Error.
A_0 760.38	A 760.40	—.02
α 718.51	a 718.47	+.04
β 686.67	B 686.71	—.04
γ 656.25	C 656.21	+.04
δ 589.95	D_1 589.51	+.44
δ_2 588.57	D_2 588.91	—.34
ε 526.74	E 526.87	—.13
β_1 518.16	b_1 518.31	—.15
β_2 517.33	b_2 517.22	+.11
β_4 516.21	b_4 516.69	—.48
φ 486.11	F 486.07	+.04
ζ 431.11	G 430.73	+.38
η 410.16	h 410.12	+.04
χ_1 396.72	H_1 396.81	—.09
χ_2 393.30	H_2 393.30	.00

In the harmonic column, $\beta_1 = \frac{64}{49}\chi_1$; β_4, ε and δ_1, are obtained by multiplying χ_2 by $\frac{21}{16}$, $\frac{75}{56}$ and $\frac{3}{2}$, respectively; $\zeta = \frac{3}{5}a$, and $\beta_2 = \frac{6}{5}\zeta$; $\eta = \frac{5}{8}\gamma$ and $\varphi = \frac{32}{27}\eta$; $\delta_2 = \frac{6}{7}\beta$. All the differences between the harmonic and the observed values are unquestionably within the limits of probable errors of observation, as indicated by the different relative estimates of lines in metallic spectra by different observers.

The influence of relative *vis viva* between the chief centres of density and of nucleation, Earth and Sun, is shown both by the spectrum and by the mean rupturing loci of the principal planetary masses, Jupiter and Saturn. Such influence is a necessary consequence of the equality of action and reaction in the interchanges of æthereal, molecular and cosmical *vis viva*. The sum, or the mean of all the gravitating accelerations upon the æthereal particles in a radius vector by each of two mutually attracting bodies, is proportioned to their respective masses, $g \propto m$. The resulting mean *vis viva* of subsidence, mg^2, is proportioned to the cubes of the masses. Nascent *vis viva* is $\pi^2 \times$ orbital *vis viva*, or $\pi^2 \times$ representative orbital projection against uniform æthereal resistance, or $2\pi \times$ the corresponding nascent projection. If we take the theoretical oscillatory ratio between the masses at the chief centres of density and nucleation, $m_0 = 331776\ m_3$; $\rho_3 = 92785700$ miles $= 149320000000$ metres. Hence the proportions:

$$149320000000 : 331776^3 :: \pi^2 : .00000000041432$$

$$149320000000 : 331776^3 :: 2\pi : .00000000065073$$

Both of these values represent spectral wave-lengths, one being about one per cent. greater than h, the other about $\frac{5}{6}$ of one per cent less than C. The ratio between the values, $\frac{\pi}{2}$, is to the ratio of C to h, $\frac{8}{5}$, as Sun's semi-diameter, 1, is to the mean rupturing radius-vector of the centre of gravity of Sun and Jupiter, 1.01859. The projectile ratio of 414.32 from h, .01014, is to the projectile ratio of C from 650.73, .00848, as Jupiter's mean projection from Sun, 5.203, is to Saturn's mean projection from Jupiter, 4.336, within $\frac{1}{3}$ of one cent.